# Theoretical Computer Science 2
## Exercise Sheet 6

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Hand in your answers to the Vips directory of the Stud.IP course until wednesday, 04.07.2024 11:59 pm. You should provide your answers either directly as PDF file or as a readable scan or photo of your handwritten notes. Submit your results as a group of four.

**Homework Exercise 1:** P **completeness [7 points]**
Consider the following problem for contextfree languages.

**Emptiness for contextfree languages** (ECFL)
**Given:** A type-2-grammar $G = \langle N, \Sigma, S, P \rangle$.
**Question:** $\mathcal{L}(G) = \varnothing$?

a) [3 points] Construct a deterministic algorithm, that decides ECFL by requiring not more than polynomial time.

b) [4 points] Show, that ECFL is P-hard wrt. LogSpace-many-one-reductions.

**Homework Exercise 2:** NP **and graph problems [6 points]**
Examine the following problems with respect to their relation with the class NP.

**Vertex Covering** (VC)
**Given:** An undirected graph $G = \langle V, E \rangle$ and $k \in \mathbb{N}$.
**Question:** Is there a set $S \subseteq V$ with $|S| = k$ and $\forall vEw : v \in S \vee w \in S$?

a) [2 points] Show that VC $\in$ NP holds, by giving a suitable nondeterministic decider, whose time complexity is bounded by some polynome.

b) [4 points] Show that SAT $\leq_m^{\log}$ VC holds by constructing a suitable reduction.
**Note:** With $k$ and a certain constellation of edges you can enforce, that each vertex covering contains exactly one from each of $k$ vertex pairs.

**Homework Exercise 3:** PSPACE **and regular languages [7 points]**
Prove Corollary 12.10 from the lecture notes:

a) [2 points] Let $A = \langle Q, \rightarrow, q_0, Q_F \rangle$ be an NFA with $\Sigma^{\leq 2^{|Q|}} \subseteq \mathcal{L}(A)$. Show, that $\mathcal{L}(A) = \Sigma^*$ is valid.

b) [5 points] Prove, that INCLUSION and EQUIVALENCE for NFAs are PSPACE-complete wrt. LogSpace many-one reductions.

**Exercise 4:**
Show that P is closed under union, concatenation, complement and Kleene operation.

**Exercise 5:**
Prove that VALIDITY is coNP-complete wrt. logspace-reductions.

| **VALIDITY** | |
|---|---|
| **Given:** | Boolean formula $\varphi$ in CNF. |
| **Question:** | Is $\varphi$ tautological, so that it hold for all assignments? |

**Exercise 6:**
Prove, that ENT is coNP-complete wrt. LogSpace-many-one-reductions.

| **ENTAILMENT** (ENT) | |
|---|---|
| **Given:** | Propositional formulas $F, F'$ in conjunctive normal form. |
| **Question:** | Does formula $F$ imply $F'$? |

**Exercise 7:**
Show by constructing an algorithm: If SAT is in P, then we could as well compute a satisfying assignment for each boolean formula in polynomial time.

**Exercise 8:**
Eine $n^2 \times n^2$ Sudoku-Matrix $M$ ist in $n^2$ viele $(n \times n)$-Blöcke unterteilt. $M$ ist korrekt ausgefüllt, wenn in jedem Block, in jeder Zeile und in jeder Spalte alle Zahlen von 1 bis $n^2$ genau einmal vorkommen. Es ist leicht zu sehen, dass SUDOKU in NP liegt, denn wir können die fehlenden Einträge raten und effizient überprüfen. Das heißt auch, dass es eine polytime-Reduktion von SUDOKU auf SAT geben muss.
Finden Sie nun solch eine Reduktion von SUDOKU auf SAT.
**Bemerkung:** Man kann sogar zeigen, dass SUDOKU NP-vollständig ist.

| **SUDOKU** | |
|---|---|
| **Given:** | Eine $n^2 \times n^2$ Sudoku-Matrix $M$ mit Einträgen in $\{1, \ldots, n^2, ?\}$ |
| **Question:** | Gibt es eine Möglichkeit die $?$-Einträge so zu ersetzen, dass ein korrekt ausgefülltes Sudoku herauskommt? |

**Exercise 9:**
Zeigen Sie, dass CLIQUE NP-vollständig bezüglich logspace-many-one-Reduktionen ist.

| **CLIQUE** | |
|---|---|
| **Given:** | Ein ungerichteter Graph $G = \langle V, E \rangle$ und eine Zahl $k \in \mathbb{N}$ |
| **Question:** | Gibt es $S \subseteq V$ mit $\lvert S \rvert = k$ und $\forall u, v \in S$: $\langle u, v \rangle \in E$? |