René Maseli
Prof. Dr. Roland Meyer

# Theoretical Computer Science 2
# Exercise Sheet 4

TU Braunschweig
Summer semester 2023

Release: 06/02/2023
Due: 06/14/2023, 18:30

Hand in your answers to the Vips directory of the Stud.IP course until wednesday, 14.06.2023 06:30 pm. You should provide your answers either directly as PDF file or as a readable scan or photo of your handwritten notes. Submit your results as a group of four. On the front page, state the **degree programme, name, surname and student id** of each member of your group.

**Homework Exercise 1: Reduction [7 points]**

Show by using a reduction, that the following problems are undecidable.

| | |
|---|---|
| **Triple-PCP** | |
| **Given:** | A finite sequence of triples $\langle x_1, y_1, z_1\rangle, \dots, \langle x_k, y_k, z_k\rangle$ of words over $\{0, 1\}$. |
| **Question:** | Does a non-empty sequence of indexes $i_1, \dots, i_n$ exist, such that $x_{i_1}, \dots, x_{i_n} = y_{i_1}, \dots, y_{i_n} = z_{i_1}, \dots, z_{i_n}$? |

a) [3 points] Show that the Triple-PCP is not co-semi-decidable.

Consider the following partial function choose : TM × TM × $\{0, 1\}^* \to \{0, 1\}$:

$$\text{choose}(\langle M_0\rangle, \langle M_1\rangle, x) = \begin{cases} 0 & \text{if } x \in \mathcal{L}(M_0) \\ 1 & \text{if } x \in \mathcal{L}(M_1) \setminus \mathcal{L}(M_0) \\ \text{undefined} & \text{if } x \notin \mathcal{L}(M_0) \cup \mathcal{L}(M_1) \end{cases}$$

b) [4 points] Show that choose is uncomputable.

**Remark**: Intuitively any Turing machine will have input words, that should be mapped to 1, but it cannot output after a finite amount of steps.

**Homework Exercise 2: Rice's Theorem [8 points]**

If possible, apply Rice's theorem on the following languages. State either a positive and a negative recusively-enumerable language, a positive Turing machine and a language-equivalent negative Turing machine, or the trivial solution.

a) [2 points] $L_1 = \{\langle M\rangle \mid M$ kehrt vor dem Akzeptieren immer zum linken Ende des Bandes zurück. $\}$

b) [2 points] $L_2 = \{\langle M\rangle \mid M$ lehnt jedes Wort mit weniger als 50 Buchstaben ab. $\}$

c) [2 points] $L_3 = \{\langle M\rangle \mid \mathcal{L}(M) = \text{SELF-ACCEPT}\}$

d) [2 points] $L_4 = \{\langle M\rangle \mid \forall x \in \{0, 1\}^* : x \in \mathcal{L}(M) \Leftrightarrow x^{reverse} \notin \mathcal{L}(M)\}$

**Homework Exercise 3: Acceptance with regular languages [5 points]**

In exercise sheet 1, we have seen that the membership problem of a fixed regular language can be decided by turing machines that can only move right and halt at the end of the input. Effectively, those machines have constant space complexity, as they can leave the input unmodified and require no working tape. The different is true, if the NFA has to be read from the input.

We define the language

$$\text{REG-ACCEPT} = \{w\#x \mid M_w \text{ describes a finite Automaton and } x \in \mathcal{L}(M_w)\}\ .$$

a) [5 points] Show REG-ACCEPT $\in L$, by describing the workings of a decider with a fixed number of logarithm-bounded working tapes.

**Remark.** You may assume, that the encoded Turing machine is deterministic.

**Exercise 4:**

If possible, apply Rice's theorem on the following languages. Reason why or why not the theorem is applicable.

$L_5 = \{w \in \{0,1\}^* \mid \mathcal{L}(M_w) \text{ ist nicht entscheidbar. }\}$

$L_6 = \{\langle M\rangle \mid \mathcal{L}(M) \leq \text{ACCEPT}\}$

$L_7 = \{\langle M\rangle \mid \text{ es gibt } n \leq |\delta_M| \text{ mit } 0^n \in \mathcal{L}(M)\}$

$L_8 = \{\langle M\rangle \mid \{0011\}.\mathcal{L}(M) = \Sigma^*\}$

**Exercise 5:**

Betrachten Sie die Sprache TOTAL = $\{\langle M\rangle \mid M \text{ ist ein Entscheider.}\}$. Warum kann der Satz von Rice hier nicht angewendet werden? Was ist der Unterschied zu Aufgabenteil (a) in der vorherigen Aufgabe 1?

Zeigen Sie per Reduktion, dass TOTAL weder semi-entscheidbar noch co-semi-entscheidbar ist.

**Hinweis.** Sie haben in der letzten Hausaufgabe bereits von einer Sprache gezeigt, dass diese weder semi-entscheidbar noch co-semi-entscheidbar ist.

**Exercise 6:**

Betrachten Sie die folgende Sprache $L_{\text{Copy}} = \{w\#w \mid w \in \{a,b\}^*\} \subseteq \{a,b,\#\}^*$.

Ordnen Sie die Sprache $L_{\text{Copy}}$ möglichst genau in die folgenden Klassen ein: $DTIME(O(f(n)))$, $NTIME(O(g(n)))$, $DSPACE(O(h(n)))$ und $NSPACE(O(j(n)))$. Findet Sie dazu möglichst kleine Funktionen $f, g, h$ und $j$, sodass $L_{\text{Copy}}$ in den jeweiligen Klassen enthalten ist.

Begründen Sie ihre Wahl, indem Sie jeweils die Arbeitsweise einer passenden Turingmaschine erklären (genaue Konstruktionen sind unnötig).