Theoretical Computer Science 1		
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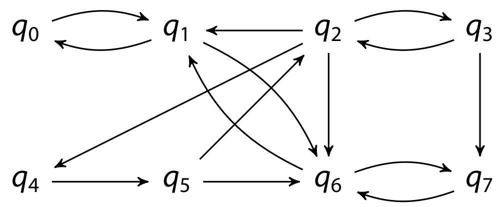
Prof. Dr. Roland Meyer		Winter semester 2022/23

Release: 15.11.2022 Due: 25.11.2022, 09:45

Hand in your solutions to the Vips directory of the StudIP course until Friday, 25.11.2022 09:45 pm. You should provide your solutions either directly as .pdf file or as a readable scan/photo of your handwritten notes. Submit your results as a group of four.

Exercise 1: Graph Reachability [7 points]

Find all vertices in the following graph $G = \langle V, E\rangle$, that are reachable from the start node q_0.

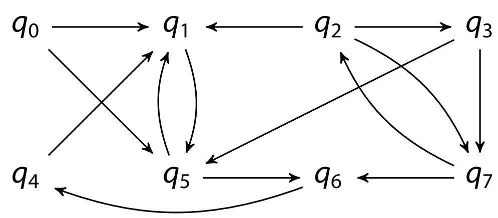


a) [3 points] Formulate a $\sqcup$-continuous function $f : \mathcal{P}(V) \to \mathcal{P}(V)$, that is suitably describes the propagation of the reachability property.

b) [4 points] Compute $\mathrm{lfp}(f)$ using the sequence in Kleene's fixed point theorem.

Exercise 2: Graph Unreachability [10 points]

Find all vertices in the following graph $G = \langle V, E\rangle$, that are **not** reachable from the start node q_0.



Consider the function $f : \mathcal{P}(V) \to \mathcal{P}(V)$.

$$f(X) = \{v \in V \mid v \neq q_0 \text{ AND } (\forall x \in V \setminus X \colon \langle x, v\rangle \notin E)\}$$

a) [3 points] Show that f is monotone in $\langle \mathcal{P}(V), \subseteq\rangle$.

b) [3 points] Show that f is $\sqcap$-continuous in $\langle \mathcal{P}(V), \subseteq\rangle$.

c) [4 points] Compute $\mathrm{gfp}(f)$ using the sequence in Kleene's fixed point theorem.

$[x := 0]^0$
while $[x^2 < y]^1$ **do**
  $[x := x + 1]^2$
end while
if $[x^2 = y]^3$ **then**
  $[z := 1]^4$
else
  $[x := y]^5$
  $[z := 0]^6$
end if
$[\text{skip}]^7$

$[x := 0]^0$
while $[x < 2^4]^1$ **do**
  $[y := 3x + 2]^2$
  while $[y < 5x]^3$ **do**
    $[y := y + 2]^4$
    if $[3x < y]^5$ **then**
      $[x := x + 1]^6$
    end if
  end while
end while
$[x := x - 14]^7$

Exercise 3: Live Variables [9 points]

Map each block in the left program to the set of variables that may be read by some other block later in the program order.

a) [1 point] Draw the control flow graph G. Note that this is a backwards analysis.

b) [3 points] Consider the lattice $\mathcal{D} = \langle \mathcal{P}(\{x, y, z\}), \subseteq \rangle$. Assign for each block $b \in B$ a suitable, monotone transfer function f_b over this lattice.

c) [5 points] Consider the data flow system $\langle G, \mathcal{D}, \{x, y, z\}, (f_b)_{b \in B} \rangle$. Write down the induced equation system and determine its least solution using Kleene's fixed point theorem.

Exercise 4: Reaching Definitions [9 points]

Map each block in the right program to the set of assignment blocks, that may have determined the current value of some variable when this block starts.

a) [1 point] Draw the control flow graph G. Mark its extremal blocks. Note that this is a forwards analysis.

b) [3 points] Consider the lattice $\mathcal{D} = \langle \mathcal{P}(\{x, y\} \times (B + \{?\})), \subseteq \rangle$. Assign for each block $b \in B$ a suitable, monotone transfer function f_b over this lattice.

c) [5 points] Consider the data flow system $\langle G, \mathcal{D}, \{(x, ?), (y, ?)\}, (f_b)_{b \in B} \rangle$. Write down the induced equation system and determine its least solution using Kleene's fixed point theorem.