

Theoretical Computer Science 1

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Exercise Sheet 4

TU Braunschweig
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Due: 15.01.2021, 17:00

Hand in your solutions per E-Mail to your tutor until Friday, 15.01.2021 17:00 pm. You should provide your solutions either directly as .pdf file or as a readable scan/photo of your handwritten notes.

Note: The last two tasks 3. and 4. will be content of the next lecture after the winter holidays.

Exercise 1: Equivalence classes [7 points]

a) [4 points] Consider the language

$$\mathcal{L} = \{a^n b^n \mid n \in \mathbb{N}\}.$$

Prove that

$$\begin{aligned} [a^n]_{\equiv_{\mathcal{L}}} &= \{a^n\} \text{ for all } n \in \mathbb{N} \\ [a^{n+1}.b]_{\equiv_{\mathcal{L}}} &= \{a^{\ell+1}.b^{\ell+1-n} \mid \ell \in \mathbb{N}, \ell \geq n\} \text{ for all } n \in \mathbb{N} \end{aligned}$$

holds.

Find all remaining equivalence classes with respect to $\equiv_{\mathcal{L}}$. In particular, for all $n, m \in \mathbb{N}$ determine the equivalence class of $a^n b^m$. (You do not have to give a formal proof.)

b) [3 points] Consider the language

$$\mathcal{L} = \{w \in \{a, b\}^* \mid w \text{ contains } aa \text{ or } bb\}.$$

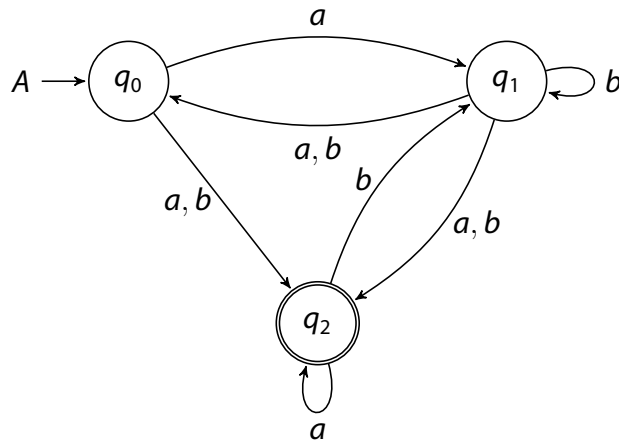
Find all equivalence classes of $\equiv_{\mathcal{L}}$.

Construct the equivalence class automaton $A_{\mathcal{L}}$.

Hint: With "contains aa " we mean that w is of the form $w = w_1.a.a.w_2$.

Exercise 2: Minimization [8 points]

Consider the following NFA A over $\{a, b\}$.



a) [2 points] From A , construct a language equivalent DFA B using the Rabin-Scott power set construction.

Make sure that B has no unreachable states.

b) [3 points] Determine the \sim -equivalence classes on the states of B by using the Table-Filling-Algorithm from the lecture.

Make clear in which order the cells of the table were marked.

c) [2 points] Give the minimal DFA C for $\mathcal{L}(A)$. Make use of the \sim -equivalence classes.

d) [1 point] Compare the size (number of states) of A , B and C .

Exercise 3: Pumping Lemma [6 points]

- a) [3 points] Consider $\Sigma = \{a, b\}$. For any word w let $|w|_a$ be the number of occurrences of symbol a in w . $|w|_b$ is defined analogously.

By using the Pumping Lemma, prove that

$$\mathcal{L} = \{w \in \Sigma^* \mid |w|_b + 7 > |w|_a\}$$

is not regular.

- b) [3 points] By using the Pumping Lemma, prove that

$$\mathcal{L} = \{w \in \Sigma^* \mid |w|_a \neq |w|_b\}$$

is not regular.

Hint to b): Think about the following: For some natural number $n \in \mathbb{N}$, which number is divisible by all numbers $\leq n$?

Exercise 4: Context free grammars [10 points]

- a) For each of the following languages, give a context free grammar that produces that language.

i) [2 points] $\mathcal{L}_1 = \{w \in \{a, b, (,)\}^* \mid w \text{ is correctly parenthesized}\}$

ii) [2 points] $\mathcal{L}_2 = \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$

iii) [2 points] $\mathcal{L}_3 = \{w \in \{a, b\}^* \mid |w|_a \neq |w|_b\}$

- b) A context free grammar G is called **regular** if it is left linear or right linear. Right linear means that the right-hand sides of all production rules contain at most one non-terminal which (if it exists) is at the right most position. Hence, all rules are of the form $X \rightarrow w$ or $X \rightarrow w.Y$ where $w \in \Sigma^*$. Left linear is defined similarly.

Prove that the regular languages exactly coincide with the languages that are produced by some right linear grammar G .

- [2 points] Explain how to construct a right linear grammar G from a given NFA A such that $\mathcal{L}(G) = \mathcal{L}(A)$ holds.
- [2 points] Explain how to construct an NFA A from a given right linear grammar G such that $\mathcal{L}(G) = \mathcal{L}(A)$ holds.

Remark: An analogous result holds for left linear grammars as well. That is why we speak of **regular** grammars in both cases.