Games with perfect information
Exercise sheet 1

Sebastian Muskalla

TU Braunschweig

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Out: April 10 Due: April 17

Submit your solutions on Wednesday, April 17, during the lecture.
You may submit in groups of two students.

Exercise 1: Nim game tree

Complete the tree from Example 2.3 from the lecture notes, i.e. draw the full tree of plays for the Nim game from the initial state $(2, 2, 1)$, where we assume that the first player is active. For every node, write down the Nim sum. Furthermore, mark all states from which the first player can enforce that she wins.

Hint: When two positions are equal up to a permutation of the piles of coins, you only need to draw one of them. For example, $(2, 0, 1)^1$ and $(0, 2, 1)^1$ are essentially the same state: The first player is active, there are two non-empty piles of coins, one with two and one with only one coin.

Exercise 2: From unbalanced to balanced Nim states

Prove Lemma 2.9 from the lecture notes: Let $(c_1, \ldots, c_k)$ be an unbalanced state. There is a direct successor (i.e. a state to which the active can go with a single move) that is balanced.

Hint: Consider the smallest index j such that $\text{Nim}\Sigma(c_1, \ldots, c_k)_j$ is odd. (Note that "smallest" means that the corresponding bit is most significant.) Prove that there is an index i with $c_{ij} = 1$ that can be modified to get to a balanced state.

Exercise 3: Three-player Nim

Consider a three player variant of Nim. The states and the rules are as in the two player variant, the only change being that three players take sequential turns, i.e. the first, then the second, then the third, then again the first player (and so on) pick a non-empty pile and remove a positive number of coins. The player that picks the last coin (and thus empties the last pile of coins) wins, the other two players lose.

Consider the initial position $(2, 1)^1$, i.e. the first player is active, there is one pile with one coin and one pile with two coins. Can any of the players enforce that she wins, no matter how the other two players move?