Exercises to the lecture
Semantics
Sheet 6

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Exercise 6.1 (Hindley-Milner Type Order)
We defined types by

$$\begin{aligned}\tau_{mono} &::= \alpha \mid C\tau_{mono}\dots\tau_{mono}\\ \tau &::= \forall\alpha.\tau\end{aligned}$$

where α is from a set of variables and C is from a set of type constructors including $\rightarrow$. We define an equivalence to sort out unnecessary types: Let $\equiv$ be the least equivalence containing for any type τ and variable $\alpha, \gamma \notin free(\tau)$, $\beta \in free(\tau)$,

$$\forall\beta_1\dots\beta_n\alpha.\tau \equiv \forall\beta_1\dots\beta_n.\tau \quad \text{and} \quad \forall\beta_1\dots\beta_n\beta.\tau \equiv \forall\beta_1\dots\beta_n\gamma.\{\beta\mapsto\gamma\}\tau$$

Further, types will only be looked at up to the above equivalence. When a type is not allowed to have quantors and it has an equivalent quantor-free type, the latter is considered. We defined a partial specialization order on types:

$$\frac{\tau' = \{\alpha_1 \mapsto \tau_1, \dots, \alpha_n \mapsto \tau_n\}\tau \qquad \beta_i \notin free(\forall\alpha_1\dots\alpha_n.\tau)}{\forall\alpha_1\dots\alpha_n.\tau \sqsubseteq \forall\beta_1\dots\beta_m.\tau'}$$

Show the Lemma from the lecture:

Lemma. *$\sqsubseteq$ is a partial order on the set of types (up to equivalence) with a least element. Downward directed sets also contain a unique minimal element.*
Bonus: *Show that it is actually a meet-semilattice (I.e. each finite subset has a meet).*