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Exercises to the lecture
Semantics
Sheet 4

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Exercise 4.1 (Properties of α, γ)
Proof the following theorem from the lecture.

(1) $(M, \subseteq)$ is a complete lattice. $(M_*, \subseteq)$ is isomorphic to $(\mathbb{P}(\Sigma^*/_\sim))$

(2) (α, γ) form a Galois connection, i.e.

$$L \subseteq \gamma(V) \Leftrightarrow \alpha(L) \subseteq V$$

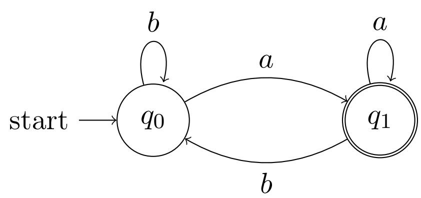
(3) $\alpha(\gamma(V)) = V$, so they are a Galois insertion

(4)
- $\alpha(L_1 \cup L_2) = \alpha(L_1) \cup \alpha(L_2)$
- $\alpha(\top) = \top$
- $\alpha(\bot) = \bot$

(5) If A is NBA, $\gamma(\alpha(L(A))) = L(A)$, thus

$$L \subseteq L(A) \Leftrightarrow \alpha(L) \subseteq \alpha(L(A)).$$

Exercise 4.2 (NBA Complementation)
Consider the NBA $\mathcal{A}$ over $\Sigma = \{a, b\}$ below:



Check $\{(a+b)^* b^\omega\} \subseteq L(A)$ and $\{a^n b^n n \in \mathbb{N}\}$.