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Exercises to the lecture
Semantics
Sheet 2

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In the lecture, we constructed a set of types from a given APTA $\mathcal{A}$, namely

$$\begin{aligned} \theta &::= q \mid \tau \to \theta \\ \tau &::= \bigwedge\{(\theta_1, m_1), \ldots, (\theta_k, m_k)\}, \end{aligned}$$

where $q \in Q$ is a state of $\mathcal{A}$ and $0 \leq k$, i.e. the conjunct may be empty.
Intuitivly, a type describes the (powerset-constructional) behaviour of $\mathcal{A}$ on any tree: A tree typed by a type q is accepted by $\mathcal{A}$ from state q. A tree t typed by a type

$$\bigwedge\{(\theta_{1,1}, m_{1,1}), \ldots, (\theta_{1,k_1}, m_{1,k_1})\} \to \ldots \to \bigwedge\{(\theta_{n,1}, m_{n,1}), \ldots, (\theta_{n,k_n}, m_{n,k_n})\} \to q$$

is an uncomplete tree, that requires n trees $s_1, \ldots, s_n$, each s_i having all types $\theta_{i,1}, \ldots, \theta_{i,k_i}$, in order to be accepted by $\mathcal{A}$ from state q. Whenever s_i is used in t, the just mentioned run of $\mathcal{A}$ visits the root of s_i in a state $q_{i,j}$ for some $1 \leq j \leq k_i$, where $\theta_{i,j} = \tau \to \ldots \to q_{i,j}$, and this path of the run from the root of t up to the root of s_i sees maximal priority $m_{i,j}$.

Exercise 2.1 (Language of APTA)
Look at the APTA $\mathcal{A} = (\Sigma, Q, \delta, q_0, \Omega)$ with $\Sigma = \{a : \sigma \to \sigma \to \sigma, b : \sigma\}$, $Q = \{q_0, q_1\}$, $\Omega = \{(q_0, 2), (q_1, 1)\}$ and transition relation

$$\begin{aligned} \delta(q_0, a) &= (1, q_1) \wedge (1, q_0) \wedge (2, q_0) \\ \delta(q_1, a) &= (1, q_1) \\ \delta(q_0, b) &= tt. \end{aligned}$$

Describe its language.

Exercise 2.2 (Well-formed types of APTAs)
Which of the following types are well-formed for the APTA $\mathcal{A}$ from Ex. 1?

1. $\bigwedge\{(q_0, 2), ((q_1, 1) \to q_0, 1)\} \to q_0$
2. $\bigwedge\{((q_0, 2) \to q_1, 2), ((q_1, 1) \to q_1, 1)\} \to \bigwedge\{(q_0, 2), (q_1, 1)\} \to q_0$
3. $\bigwedge\{(q_0, 2), (q_1, 1), (q_1, 2)\} \to q_0$
4. $\bigwedge\{(q_2, 2)\} \to q_0$

List the atomic types θ with $\theta ::_a \sigma \to \sigma$.

Exercise 2.3 (Type Judgements)
Set the types

$$\begin{aligned}
\theta_{x,q_0} &= \bigwedge\{(q_0, 2), (q_1, 2)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_0, \\
\theta_{x,q_1} &= \bigwedge\{(q_0, 2), (q_1, 1)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_1, \\
\theta_{H,q_0} &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \theta_{x,q_0}, \\
\theta_{H,q_1} &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \theta_{x,q_1}, \\
\theta_F &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_0.
\end{aligned}$$

Show a formal deduction for the type judgement

$$\{H : (\theta_{H,q_0}, 2)^f, H : (\theta_{H,q_1}, 2)^f, F : (\theta_F, 2)^f\} \vdash \lambda x \lambda y.a(xyy)(F(Hx)y) : \theta_F.$$