
In-class Exercises to the Lecture Logics
Sheet 4

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Exercise 4.1 [Resolution]

Prove by resolution that the following formula is unsatisfiable:

$$(\neg p \vee s) \wedge (p \vee q \vee \neg r) \wedge (r \vee q \vee s) \wedge (q \vee \neg s) \wedge (\neg q \vee s) \wedge (\neg q \vee \neg s)$$

Exercise 4.2 [Formulae in 2CNF]

A formula is in 2CNF if it is of the form $C_1 \wedge \dots \wedge C_n$, where each C_i is a clause with at most two literals. Devise a method to check satisfiability for such formulae that runs in polynomial time.

Exercise 4.3 [Complete sets of connectives]

- a) Prove that for any formula $A \in F(\{\vee\})$, we have $\varphi(A) = 0$, where φ is the valuation with $\varphi(p) = 0$ for each atomic formula p .
- b) Show that $\{\vee\}$ is *not* a complete set of connectives.

Exercise 4.4 [Predicate Logic]

Let $A \equiv \forall x(p_{\text{CanSwim}}(x) \rightarrow p_{\text{IsFish}}(x))$.

- a) Present an Interpretation I with $I \models A$.
- b) Present an Interpretation $I = (D, I_c, I_v)$ with $I \models A$, where $D = \{\text{Duck, Herring, Carp}\}$.