

Exercises to the lecture Logics
Sheet 7

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Due 24.07.2012 12:00 Uhr

Please note: The points on this sheet are not counted towards the 100% of which you need at least 60%.

Achtung: Die Punkte auf diesem Blatt zählen nicht zu den 100%, von denen mindestens 60% erreicht werden müssen.

Exercise 7.1 [Resolution]

Using resolution, show that the formula

$$\forall z_1[q(z_1)] \vee \neg \forall x[(q(x) \vee r(x)) \wedge \exists z_2[\neg p(z_2) \wedge (p(z_2) \vee \neg r(x))]]$$

is a tautology. This amounts to

- a) negating the formula,
- b) bringing the result into clause form, and
- c) applying resolution to the formula in clause form.

Exercise 7.2 [Calculating MGU]

For each of the following sets of literals, decide whether it is unifiable and if so, determine an MGU.

- a) $\{q(x, z), q(h(y, z), f(a)), q(h(f(b), z), z)\}$.
- b) $\{p(x, f(y)), p(f(a), y)\}$.

Exercise 7.3 [Equality in predicate logic]

Show that for each formula A in predicate logic, we can construct a formula A' that does not contain “=” and is equisatisfiable to A . (Formulae A and B are called equisatisfiable if A is satisfiable if and only if B is satisfiable.)

Exercise 7.4 [An application to graphs]

By a *graph*, we mean an undirected (not necessarily finite) graph that may have loops. The *length* of a path is the number of edges in the path. The *distance* between two vertices is the length of a shortest path between them. We call a vertex *isolated* if there is no other vertex to which it is connected by an edge.

- a) Formalize the following statement as a formula in first order predicate logic: *If for each pair of vertices, their distance is at most 2 and there are at least two distinct vertices, then there is no isolated vertex.*
- b) Negate the formula and transform the result into an equisatisfiable formula without “=”.
- c) Using resolution, show that the obtained formula is unsatisfiable.

Delivery: until 24.07.2012 12:00 Uhr into the box next to room 34/401.4