Games with perfect information
Exercise sheet 5

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Out: May 2 Due: May 9

Submit your solutions on Wednesday, May 9, at the beginning of the lecture.
Please submit in groups of three persons.

Exercise 1: Encoding winning conditions

Let $G = (V_\square \uplus V_\bigcirc, R)$ be a deadlock-free, finite game arena. Let $x, y \in V$ be two positions, $x \neq y$.

a) Present a reachability/safety game whose winning condition encodes the following property: A play is won by the universal player if it does not visit both x and y.

b) Present a parity game whose winning condition encodes the following property: A play is won by the existential player if it visits x at least once, and later visits y infinitely often.

c) Present a parity game whose winning condition encodes the following property: A play is won by the existential player if it either does not visit x infinitely often, or it visits both x and y infinitely often.

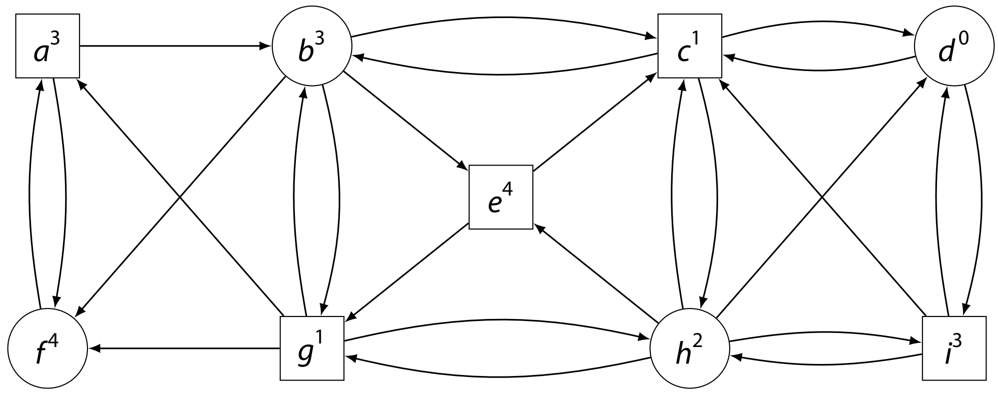
d) Present a parity game whose winning condition encodes the following property: A play is won by the existential player if it either does not visit x infinitely often, or it visits x, but not y infinitely often.

For each part, reason briefly why your construction is correct.

Note: You are allowed to modify the game arena G if needed.

Exercise 2

Consider the parity game given by the following graph. For each vertex labeled with x^i, the letter x denotes the name of the vertex, the superscript denotes its priority $\Omega(x) = i$.



For each player, identify her winning region and present a uniform positional winning strategy. Reason briefly why the strategies are indeed winning.

Exercise 3: Uniform winning strategies I

Prove Part a) of Lemma 6.5 from the lecture notes, including all technical details:

Let $x, x' \in V$ be positions such that player $\largestar \in \{\bigcirc, \square\}$ has positional winning strategies $s_{\largestar,x}, s_{\largestar,x'}$ winning from x resp. x'. Then there is a positional strategy $s_{\largestar}$ that is winning from both x and x'.

Exercise 4: Uniform winning strategies II

Prove Part b) of Lemma 6.5 from the lecture notes:

Let X be a set of positions such that for each $x \in X$, $\largestar \in \{\bigcirc, \square\}$ has a positional strategy $s_{\largestar,x}$ that is winning from x. Then there is a positional strategy $s_{\largestar}$ that is uniformly winning from all positions $x \in X$.

Hint: A proof by induction will not work, since X may be infinite. Note that we assumed that V is countable, this in particular means that we can write $V = \{v_0, v_1, v_2, \ldots\}$ for appropriately chosen v_i. Many of the arguments from Exercise 3 can be reused.