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Exercises to the lecture
Concurrency Theory
Sheet 7

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Exercise 7.1 (Simulation Relation)
Let $TS = (\Gamma, \gamma_0, \rightarrow)$ be a transition system and $\leq \subseteq \Gamma \times \Gamma$ be a quasi ordering. Show that $\leq$ is a simulation relation if and only if for each upward closed set $I \subseteq \Gamma$ we have that $\text{pre}(I)$ is also upward closed.

Exercise 7.2 (Product of WSTSs)
Let $TS^1 = (\Gamma^1, \gamma_0^1, \rightarrow^1, \leq^1)$ and $TS^2 = (\Gamma^2, \gamma_0^2, \rightarrow^2, \leq^2)$ be WSTSs. We define their product $TS^1 \times TS^2$ to be $(\Gamma, \gamma_0, \rightarrow, \leq)$ with

- $\Gamma = \Gamma^1 \times \Gamma^2$,
- $\gamma_0 = (\gamma_0^1, \gamma_0^2)$,
- $(\gamma^1, \gamma^2) \rightarrow (\bar{\gamma}^1, \bar{\gamma}^2)$ if $\gamma^1 \rightarrow^1 \bar{\gamma}^1$ and $\gamma^2 \rightarrow^2 \bar{\gamma}^2$,
- $(\gamma^1, \gamma^2) \leq (\bar{\gamma}^1, \bar{\gamma}^2)$ if $\gamma^1 \leq^1 \bar{\gamma}^1$ and $\gamma^2 \leq^2 \bar{\gamma}^2$.

Prove that $TS^1 \times TS^2$ is a WSTS.

Exercise 7.3 (Termination)
Let $TS = (\Gamma, \gamma_0, \rightarrow, \leq)$ be a WSTS where

- $\gamma \leq \gamma'$ is decidable for each $\gamma, \gamma' \in \Gamma$,
- the set $\text{post}(\gamma) = \{\gamma' | \gamma \rightarrow \gamma'\}$ is finite and computable, for each $\gamma \in \Gamma$.

The WSTS TS is called *terminating* if every computation starting in γ_0 is finite.

Show that the *termination problem* is decidable. That is, given a WSTS TS like above, decide whether TS is terminating.

Exercise 7.4 (LCSs with strong messages)
Let $L_S = (Q, q_0, \{c\}, M \cup S, \rightarrow)$ be a lossy channel system with a single channel c and S a finite set of *strong messages*. This means that messages from S cannot be forgotten.

a) Define the transition relation $\rightarrow_S$ induced by L_S. It should correspond to the usual one defined in the lecture in the case $S = \emptyset$.

   Hint: You have to define a particular order that does not delete the symbols of S and acts like the subword order on M.

b) Assume there is a $k \in \mathbb{N}$ such that the number of strong messages in each channel is bounded by k. Prove that the resulting restriction $L_S(k)$ of L_S is a WSTS.

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