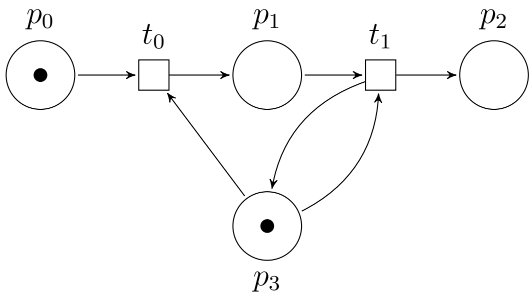
Roland Meyer, Elisabeth Neumann Winter term 2017/18

Out: November 29 Due: December 5

Submit your solutions until Wednesday, December 6, 12:00 am. You may submit in groups up to three persons.

Exercise 1: The marking equation

Consider the following Petri net.



a) Write down the connectivity matrix $\mathbb{C}$ of the Petri net.

b) Argue that the marking $M_f = (0, 0, 1, 0)$ that has one token in p_3 is not reachable from the initial marking $M_0 = (1, 0, 0, 1)$.

c) Prove that the marking equation $M_f - M_0 = \mathbb{C} \cdot c$ has a solution (i.e. there is a vector $c \in \mathbb{N}^T$ satisfying the equation).

Exercise 2: Petri net constructions

a) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M_0', M_f') with $M_0'(p) = 0$ for all places but a single place p' with $M_0'(p') = 1$ and $M_f'(p) = 0$ for all places such that
$M_f \in R(N, M_0)$ iff $M_f' \in R(N', M_0')$.

b) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M_0', M_f') such that
M_f is coverable from M_0 in N iff M_f' is reachable from M_0' in N'.

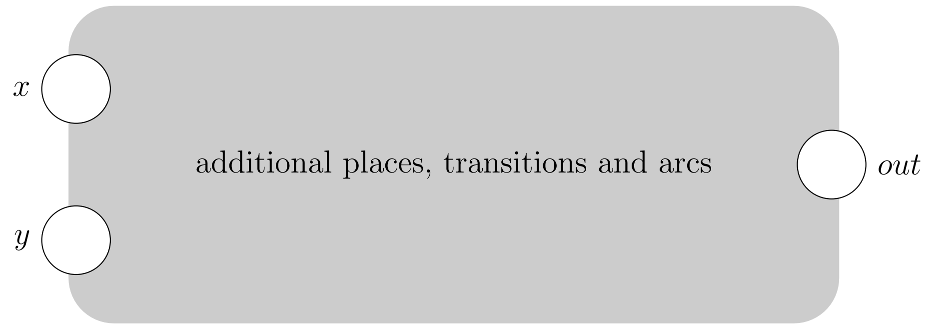
c) Construct a Petri net N with only 3 places, a marking M_0 and markings $M_{c\wedge r}, M_{\neg c\wedge\neg r}$ and $M_{c\wedge\neg r}$ such that

- $M_{c\wedge r}$ is reachable and coverable from M_0,
- $M_{\neg c\wedge\neg r}$ is neither reachable nor coverable, and
- $M_{c\wedge\neg r}$ is coverable, but not reachable.

In each part of this exercise, argue briefly that your construction is correct.

Exercise 3: Addition and multiplication

Consider the (incomplete) Petri net containing places x, y and *out* depicted below.



a) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) = m + n$.

   (*Terminating* means that no transition is enabled anymore.)

b) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) \in \{0, \dots, m \cdot n\}$.

In each part of this exercise, argue briefly that your construction is correct.