Concurrency theory
Exercise sheet 1

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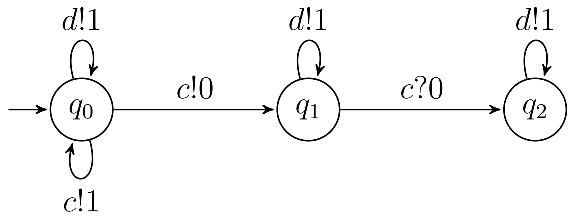
Out: November 22 | Due: November 28

Submit your solutions until Wednesday, November 28 12:00 am. You may submit in groups up to three persons.

Exercise 1: Expand, Enlarge and Check

Consider the following lossy channel system LCS:



together with $\Gamma = \{(q_0, \varepsilon), (q_1, \varepsilon), (q_2, \varepsilon)\}$ and limit domains

$$L_0 = \left\{\top, \left(q_0, \begin{pmatrix}1^* \\ \varepsilon\end{pmatrix}\right), \left(q_0, \begin{pmatrix}\varepsilon \\ 1^*\end{pmatrix}\right), \left(q_1, \begin{pmatrix}(0+1)^* \\ 0^*.1^*\end{pmatrix}\right), \left(q_1, \begin{pmatrix}(0+1)^* \\ 1^*.0^*\end{pmatrix}\right)\right\}$$

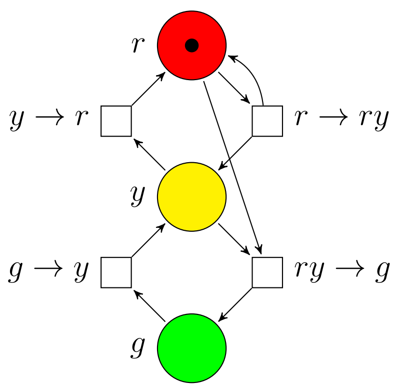
$$L_1 = L_0 \cup \left\{\left(q_0, \begin{pmatrix}1^* \\ 1^*\end{pmatrix}\right), \left(q_1, \begin{pmatrix}1^*.(0+\varepsilon) \\ 1^*\end{pmatrix}\right), \left(q_2, \begin{pmatrix}\varepsilon \\ 1^*\end{pmatrix}\right)\right\}.$$

a) Compute $Over(LCS, \Gamma, L_0)$. Provide an execution tree.

b) Compute $Over(LCS, \Gamma, L_1)$. Argue why configuration $\left(q_2, \begin{pmatrix}1 \\ \varepsilon\end{pmatrix}\right)$ is not coverable.

Exercise 2: Traffic lights and Petri nets

Consider the Petri net given by the following graphic representation.



a) Write down the net as a tuple $N = (P, T, i, o)$.

b) The net should model a traffic light, but it contains a bug and exhibits unwanted behavior. Show a valid firing sequence (from the initial marking indicated in the graphic representation) reaching a bad marking.

Modify the net to fix the problem. The resulting net should be 1-safe.

c) Model two traffic lights handling a road crossing by using two such Petri nets.

Exercise 3: The Ackermann function

a) The three-argument Ackermann function φ is defined recursively as follows.

$$\begin{aligned}
&\varphi\colon \mathbb{N}^3 \to \mathbb{N} \\
\varphi(m,n,0) &= m+n \\
\varphi(m,0,1) &= 0 \\
\varphi(m,0,2) &= 1 \\
\varphi(m,0,x) &= m && \text{for } x>2 \\
\varphi(m,n,x) &= \varphi(m,\varphi(m,n-1,x),x-1) && \text{for } n>0 \text{ and } x>0
\end{aligned}$$

Formally prove the following equalities (e.g. using induction):

$$\varphi(m,n,0)=m+n, \qquad \varphi(m,n,1)=m\cdot n, \qquad \varphi(m,n,2)=m^n.$$

b) Nowadays, one usually considers the following two-parameter variant.

$$\begin{aligned}
&A\colon \mathbb{N}^2 \to \mathbb{N} \\
A(0,n) &= n+1 \\
A(m,0) &= A(m-1,1) && \text{for } m>0 \\
A(m,n) &= A(m-1,A(m,n-1)) && \text{for } m>0 \text{ and } n>0
\end{aligned}$$

For example, we have

$$A(1,2)=A(0,A(1,1))=A(0,A(0,A(1,0)))=A(0,A(0,A(0,1)))=A(0,A(0,2))=A(0,3)=4\,.$$

Similar to this computation, write down a full evaluation of $A(2,3)$.