Concurrency theory
Exercise sheet 2

TU Braunschweig

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Winter term 2018/19

Out: November 01 | Due: November 07

Submit your solutions until Wednesday, November 07, 12:00 am. You may submit in groups up to three persons.

Exercise 1: Composition of WSTS

Consider two WSTS $TS_1 = (\Gamma_1, \rightarrow_1, \gamma_0, \leqslant_1)$ and $TS_2 = (\Gamma_2, \rightarrow_2, \bar{\gamma}_0, \leqslant_2)$. We define their composition to be $TS_1 \otimes TS_2 = (\Gamma, \rightarrow, \gamma, \leqslant)$ where

- $\Gamma = \Gamma_1 \times \Gamma_2$
- $(\gamma, \bar{\gamma}) \rightarrow (\gamma', \bar{\gamma}')$ iff $\gamma \rightarrow_1 \gamma'$ and $\bar{\gamma} \rightarrow_2 \bar{\gamma}'$
- $\gamma = (\gamma_0, \bar{\gamma}_0)$
- $(\gamma, \bar{\gamma}) \leqslant (\gamma', \bar{\gamma}')$ iff $\gamma \leqslant_1 \gamma'$ and $\bar{\gamma} \leqslant_2 \bar{\gamma}'$

Prove that $TS_1 \otimes TS_2$ is also a WSTS.

Exercise 2: Well quasi orderings

Prove or disprove that $(Bin, \leqslant)$ is a well-quasi ordering, here Bin represents set of all binary numbers $Bin = \{0, 1\}^*$ and $\leqslant$ is the lexicographing ordering with $0 \leqslant 1$.

Exercise 3: Downward closed sets

Prove that for any wqo $(A, \leqslant)$ and for every infinite decreasing sequence $D_0 \supseteq D_1 \supseteq D_2 \supseteq \ldots$ of downward closed sets, there is a $k \in \mathbb{N}$ such that $D_k = D_{k+1}$

Exercise 4: WSTS

Given a wsts $(\Gamma, \rightarrow, \gamma_0, \leqslant)$, describe an algorithm to decide if every run from γ_0 is terminating or not. Assume the wsts to be finitely branching, i.e., for every configuration $\gamma_1 \in \Gamma$ there are finitely many $\gamma_2 \in \Gamma$ with $\gamma_1 \rightarrow \gamma_2$. Prove correctness of your algorithm.