Concurrency theory
Exercise sheet 1

Sebastian Muskalla, Prakash Saivasan
TU Braunschweig

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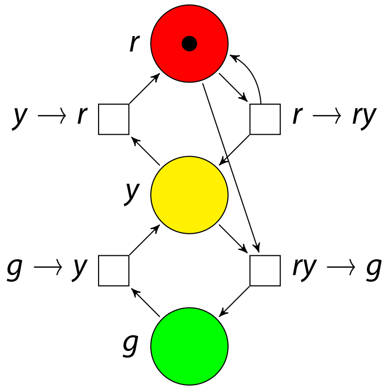
Out: October 18 Due: October 24

Submit your solutions until Tuesday, October 24, during the lecture. You may submit in groups up to three persons.

Exercise 1: Traffic lights and Petri nets

Consider the Petri net given by the following graphic representation.



a) Write down the net as a tuple $N = (P, T, i, o)$.

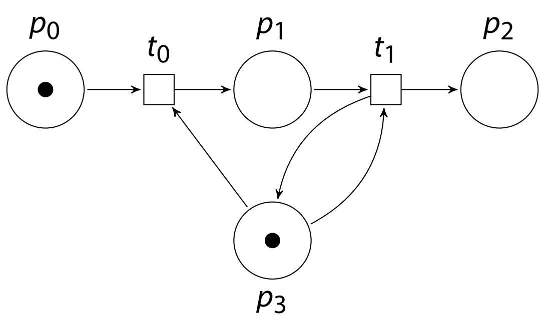
b) The net should model a traffic light, but it contains a bug and exhibits unwanted behavior. Show a valid firing sequence (from the initial marking indicated in the graphic representation) reaching a bad marking.

Modify the net to fix the problem. The resulting net should be 1-safe.

c) Model two traffic lights handling a road crossing by using two such Petri nets.

Exercise 2: The marking equation

Consider the following Petri net.



a) Write down the connectivity matrix $\mathbb{C}$ of the Petri net.

b) Argue that the marking $M_f = (0, 0, 1, 0)$ that has one token in p_3 is not reachable from the initial marking $M_0 = (1, 0, 0, 1)$.

c) Prove that the marking equation M_f - $M_0 = \mathbb{C} \cdot c$ has a solution (i.e. there is a vector $c \in \mathbb{N}^T$ satisfying the equation).

Exercise 3: The Ackermann function

a) The three-argument Ackermann function φ is defined recursively as follows.

$$
\begin{aligned}
&\varphi : \mathbb{N}^3 \to \mathbb{N} \\
&\varphi(m, n, 0) &=& \; m + n \\
&\varphi(m, 0, 1) &=& \; 0 \\
&\varphi(m, 0, 2) &=& \; 1 \\
&\varphi(m, 0, x) &=& \; m && \text{for } x > 2 \\
&\varphi(m, n, x) &=& \; \varphi(m, \varphi(m, n - 1, x), x - 1) && \text{for } n > 0 \text{ and } x > 0
\end{aligned}
$$

Formally prove the following equalities (e.g. using induction):

$$\varphi(m, n, 0) = m + n, \qquad \varphi(m, n, 1) = m \cdot n, \qquad \varphi(m, n, 2) = m^n.$$

b) Nowadays, one usually considers the following two-parameter variant.

$$
\begin{aligned}
&A : \mathbb{N}^2 \to \mathbb{N} \\
&A(0, n) &=& \; n + 1 \\
&A(m, 0) &=& \; A(m - 1, 1) && \text{for } m > 0 \\
&A(m, n) &=& \; A(m - 1, A(m, n - 1)) && \text{for } m > 0 \text{ and } n > 0
\end{aligned}
$$

For example, we have

$$A(1, 2) = A(0, A(1, 1)) = A(0, A(0, A(1, 0))) = A(0, A(0, A(0, 1))) = A(0, A(0, 2)) = A(0, 3) = 4\,.$$

Similar to this computation, write down a full evaluation of $A(2, 3)$.