Concurrency Theory (WS 2016) Out: Thu, 26 Jan Due: Wed, 01 Feb

Exercise Sheet 12

D'Osualdo, Lederer, Schneider Technische Universität Kaiserslautern

Problem 1: Petri Nets in ν-free CCS

We call a *Petri net counter* a counter that can be incremented, decremented when non-zero but cannot be tested for zero. A sequential specification for such a counter is

$$\mathsf{PNC}_0 := inc.\mathsf{PNC}_1$$
$$\mathsf{PNC}_{n+1} := inc.\mathsf{PNC}_{n+2} + dec.\mathsf{PNC}_n$$

Implement a Petri net counter in $\boldsymbol{\nu}$-free CCS using finitely many definitions.

Problem 2: Polyadic and Monadic π-calculus

The polyadic π-calculus allows tuples of any size to be sent as messages. The monadic π-calculus instead requires that all the prefixes are of the form $\overline{x}\langle y\rangle$, $x(y)$ or $\boldsymbol{\tau}$.

Your task is to show how polyadic prefixes can be implemented in the monadic π-calculus. Implementing a single polyadic action using a sequence of steps is acceptable.

Explain what happens to your encoding when you translate a process where the usage of channels is not uniform, for example $\overline{x}\langle a, b\rangle.Q \parallel x(y_1, y_2, y_3).Q'$.

Formally, for a term P in the polyadic calculus, you have to provide a translation $\mathcal{M}[\![P]\!]$ in the monadic calculus that has the same behaviour up to some $\boldsymbol{\tau}$ steps. Your translation only needs to be correct when channels are used uniformly. Argue informally why the translation is correct.

Problem 3: The Rumour Mill (aka Gossip Protocols)

Have you heard of the rumour mill of the bar "Hannenfass"? Well, the visitor of the Hannenfass —until it burnt to the ground— could be divided into two groups: The naïve ones (N) who did not know about the secret and the wise ones (W) who were introduced to the secret. No one actually knows who was the first wise person, but since everybody was either curious about the secret or wanted to brag about knowing it, the rumour mill started stirring. People went to the barman (B) to ask for a well secluded table for two where you could not be overheard. Assigned a table, they would sit and wait for some other client to join them. Of course, the naïve ones hoped to be introduced to the holy circle of the wise visitors and the latter hoped to be able to show off their knowledge. But as life goes, your wishes are not always fulfilled and from time to time a wise and an naïve visitor would not get a table together. This was mainly the fault of the barman who simply couldn't distinguish the two groups. He was already happy when his unlucky guests would order beer in frustration.

a) Complete the following model of the Hannenfass' rumour mill with initial configuration

$$\boldsymbol{\nu} s.\mathsf{W}[\,ask, s\,] \parallel \underbrace{\mathsf{N}[\,ask\,] \parallel \cdots \parallel \mathsf{N}[\,ask\,]}_{n} \parallel \boldsymbol{\nu} t.\mathsf{B}[\,ask, t\,]$$

and definitions

$$\begin{aligned} \mathsf{B}[\,ask, t\,] &:= \overline{ask}\langle t\rangle.\overline{ask}\langle t\rangle.\boldsymbol{\nu} t'.\mathsf{B}[\,ask, t'\,] \\ \mathsf{W}[\,ask, s\,] &:= \ldots \\ \mathsf{N}[\,ask\,] &:= \ldots \end{aligned}$$

Try implementing the visitors so that the bar is always busy (i.e. it cannot deadlock) and that there is a reachable state where every visitor became wise.

b) Draw the communication topology for some interesting (i.e. not completely dumb) configuration reachable from the Hannenfass' rumour mill with $n = 7$.

c) What changes in the communication topology if we restrict the ask name at the top level, i.e. when the initial configuration is

$$\boldsymbol{\nu} ask.(\boldsymbol{\nu} s.\mathsf{W}[\,ask, s\,] \parallel \underbrace{\mathsf{N}[\,ask\,] \parallel \cdots \parallel \mathsf{N}[\,ask\,]}_{n} \parallel \boldsymbol{\nu} t.\mathsf{B}[\,ask, t\,])$$