Exercise Sheet 10

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Problem 1: Strong Bisimulation is a Process Congruence

In the lecture, we defined *process congruence* as any equivalence relation $\cong$ such that for every *elementary* context $C[\,]$ we have that if $P \cong Q$ then $C[P] \cong C[Q]$. An easy consequence is that then the same is true for *any* context.

We now want to show that strong bisimulation is a process congruence. To prove this claim, we have to show that if $P \sim Q$ then

$\boxed{1}$ $\alpha.P + M \sim \alpha.Q + M$

$\boxed{2}$ $\nu a.P \sim \nu a.Q$

$\boxed{3}$ $P \parallel R \sim Q \parallel R$

$\boxed{4}$ $R \parallel P \sim R \parallel Q$

Prove $\boxed{3}$ by showing that the relation $\mathscr{S} := \left\{ \left(A \parallel C, B \parallel C\right) \;\middle|\; A \sim B \right\}$ is a bisimulation. Then pick $\boxed{1}$ or $\boxed{2}$ and prove it using a similar argument. Note that $\boxed{4}$ follows from $\boxed{3}$ and the fact that structural congruence is a strong bisimulation.

The fact that bisimulation is a congruence is important: it gives formal meaning to the claim that no environment can tell the difference between two bisimilar processes, by modelling the environment as a context.

Problem 2: Algebraic Properties of Bisimulation

a) Show that $\boldsymbol{\nu} a.(a.P) \sim \mathbf{0}$ for any P.

b) Show that $\boldsymbol{\nu} c.(a.c.P \parallel b.\overline{c}.Q) \sim \boldsymbol{\nu} c.(a.c.Q \parallel b.\overline{c}.P)$ for any P, Q.

c) Assume that the three CCS processes P, Q and R have a free name $done$ and perform an action $\overline{done}$ just before terminating.

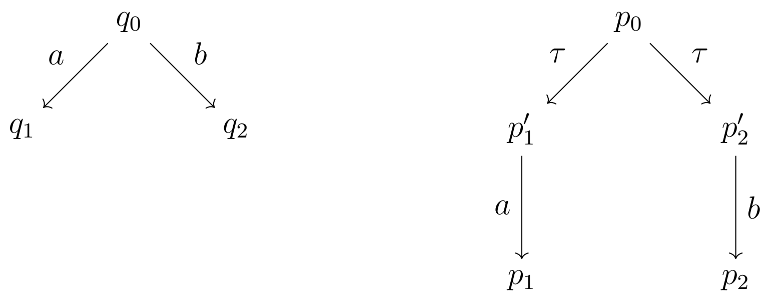
   We define the *sequential composition* of processes A and B as

   $$A; B := \boldsymbol{\nu} start.(A[\,start/done\,] \parallel start.B).$$

   Show that sequential composition is associative, i.e. $(P; Q); R \sim P; (Q; R)$.

Problem 3: Weak Simulation

Consider the following LTS:



Show that

a) q_0 weakly simulates p_0,

b) p_0 weakly simulates q_0, but

c) q_0 is not weakly bisimilar to p_0.

Problem 4: Counter II — THE REVENGE

In class we have seen the sequential specification of a counter:

$$\begin{aligned} \mathsf{Count}_0 &:= \mathit{inc}.\mathsf{Count}_1 + \overline{\mathit{zero}}.\mathsf{Count}_0 \\ \mathsf{Count}_{n+1} &:= \mathit{inc}.\mathsf{Count}_{n+2} + \mathit{dec}.\mathsf{Count}_n \end{aligned}$$

Now we give a new implementation. Let $\vec{x}_i = \mathit{inc}_i, \mathit{dec}_i, \mathit{zero}_i$:

$$\begin{aligned} \mathsf{Z}[\,\vec{x}_1\,] &:= \mathit{inc}_1.\boldsymbol{\nu}\vec{x}_2.(\mathsf{S}[\,\vec{x}_1, \vec{x}_2\,] \parallel \mathsf{Z}[\,\vec{x}_2\,]) + \overline{\mathit{zero}}_1.\mathsf{Z}[\,\vec{x}_1\,] \\ \mathsf{S}[\,\vec{x}_1, \vec{x}_2\,] &:= \mathit{inc}_1.\boldsymbol{\nu}\vec{x}_3.\big(\mathsf{S}[\,\vec{x}_1, \vec{x}_3\,] \parallel \mathsf{S}[\,\vec{x}_3, \vec{x}_2\,]\big) + \mathit{dec}_1.\big(\overline{\mathit{dec}_2}.\mathsf{S}[\,\vec{x}_1, \vec{x}_2\,] + \mathit{zero}_2.\mathsf{Z}[\,\vec{x}_1\,]\big) \end{aligned}$$

Your task is to prove that it is a correct implementation, that is $\mathsf{Count}_0 \approx \mathsf{Z}[\,\mathit{inc}, \mathit{dec}, \mathit{zero}\,]$.