Concurrency Theory (WS 2011/12)	Out: Tue, Nov 29 Due: Mon, Dec 5

Exercise Sheet 7

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Problem 1: Easy Lemma/Theorem Proofs

Let $N = (S, T, W, M_0)$. Prove the following statements:

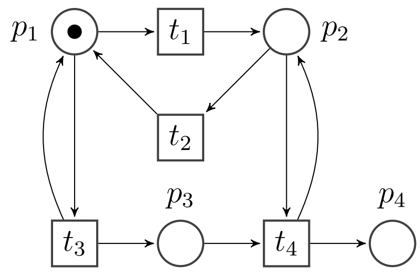
(a) If $M_0[\sigma\rangle M$ then there is a σ-labeled path $M_0 \xrightarrow{\sigma} M_\omega$ in $Cov(N)$ with $M_\omega \geq M$.

(b) Marking $M \in \mathbb{N}^S$ is coverable in N iff there is $M_\omega \in Cov(N)$ with $M_\omega \geq M$.

(c) Place $s \in S$ is unbounded iff there is $M_\omega \in Cov(N)$ with $M_\omega(s) = \omega$.

Problem 2: Coverability Graph and Place Unboundedness

Construct the coverability graph for the following Petri net:



(a) Name the unbounded places in the net by specifying all nodes in the coverability graph which allow you to deduce their unboundedness.

(b) For each of the following markings of the Petri net:

$$(1\,0\,0\,0)^T,\ (0\,0\,1\,0)^T,\ (1\,1\,0\,0)^T,\ (0\,0\,1\,1)^T,\ (1\,0\,1\,0)^T,\ (0\,1\,0\,1)^T$$

specify all the nodes in the coverability graph (if any) that cover them.

Problem 3: Proof of Lemma - t *introduces new* ω*'s*

Let $N = (S, T, W, M_0)$ be a Petri net with coverability graph $Cov(N) = (V, E, M_0)$ and let $M_0 \xrightarrow{\sigma} M_\omega^n \xrightarrow{t} M_\omega^{n+1}$ for some $\sigma \in T^n$ and $t \in T$.

Assume that for all $k \in \mathbb{N}$ there exists $M \in R(N)$ such that

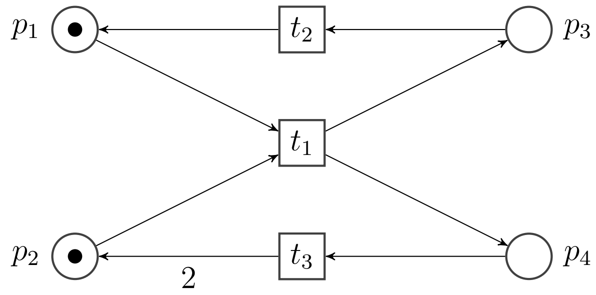
$$\begin{cases} M(s) \geq k & \text{if } s \in \Omega(M_\omega^n) \\ M(s) = M_\omega^n(s) & \text{if } s \in S \setminus \Omega(M_\omega^n). \end{cases}$$

Fix $k_0 \in \mathbb{N}$ and assume $|\Omega(M_\omega^{n+1})| = |\Omega(M_\omega^n)| + 1$. Prove that there is $M' \in R(N)$ such that

$$\begin{cases} M'(s) \geq k_0 & \text{if } s \in \Omega(M_\omega^{n+1}) \\ M'(s) = M_\omega^{n+1}(s) & \text{if } s \in S \setminus \Omega(M_\omega^{n+1}). \end{cases}$$

Problem 4: Coverability Graph and Net Boundedness

Consider the Petri net below:



(a) Use the Karp and Miller algorithm to construct the coverability graph.

(b) Describe in words an algorithm that takes a Petri net $N = (S, T, W, M_0)$ and returns the *optimal bound* $b \in \mathbb{N}_\omega$ *for the token count on all places*. Argument termination and correctness.

To be precise, the algorithm should
- return $b = \omega$, if N is unbounded
- return the smallest $b \in \mathbb{N}$ so that $M(p) \leq b$ for all $M \in R(N)$ and all $p \in S$, otherwise.