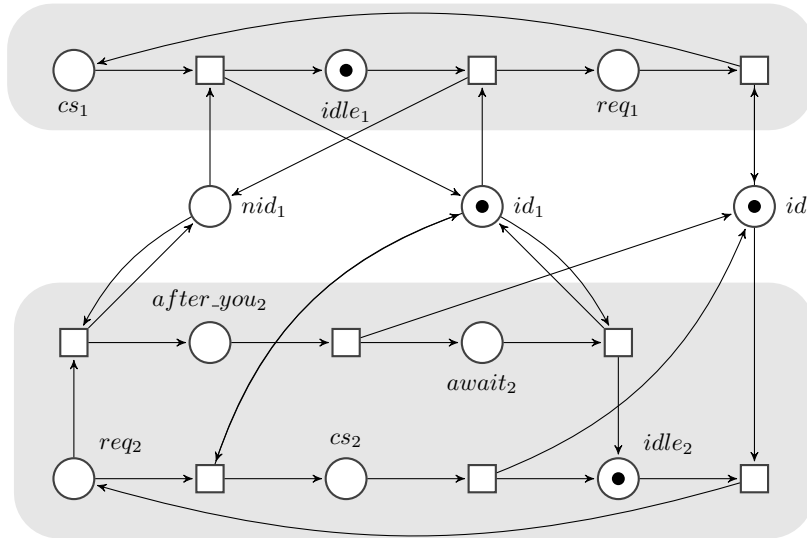


Exercise Sheet 5

Problem 1: Lamport's Mutual Exclusion Algorithm

Consider the Petri net below, describing Lamport's 1-bit mutual exclusion algorithm.



(a) set up the colinear property one would want the mutex to satisfy and determine the connectivity and trap matrices of the given Petri net;

(b) prove that the *basic verification system* is feasible;

(c) prove that the *enhanced verification system* is infeasible.

How do you interpret the fact that *bvs* is feasible and *evs* infeasible?

The exercise is a bit of work but demonstrates the power of the analysis technique. The example is taken from J. Esparza, S. Melzer: Verification of Safety Properties Using Integer Programming: Beyond the State Equation. Formal Methods in System Design 16(2): 159-189 (2000).

Problem 2: Conflict vs. Causality vs. Concurrency

Prove that for occurrence net (B, E, G) and distinct $x, y \in B \cup E$, precisely one of the following holds:

- x and y are causally related (i.e. $x < y$ or $x > y$)
- x and y are in conflict (i.e. $x \# y$)
- x and y are concurrent (i.e. $x \text{ co } y$)

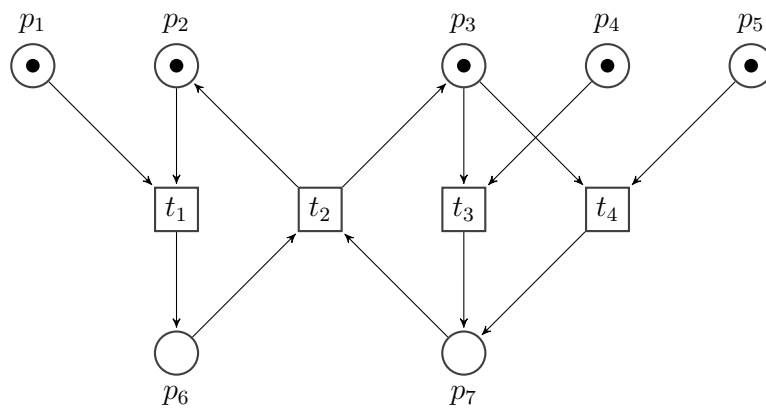
Problem 3: Configurations and Firing Sequences

Let $C = \{e_1, \dots, e_n\}$ be a configuration with normal event ordering: $i < j$ if $e_i < e_j$. Prove that $e_1 \dots e_n$ is a possible firing sequence in the unfolding, producing the marking $(\{e_\perp\}^\bullet \cup C^\bullet) \setminus C$.

Problem 4: Unfoldings, Configurations, and Cuts

Consider the Petri net depicted below:

- (a) unfold the Petri net and outline one of its prefixes;
- (b) describe/list all the configurations and cuts of your unfolding.



Why is your unfolding finite?