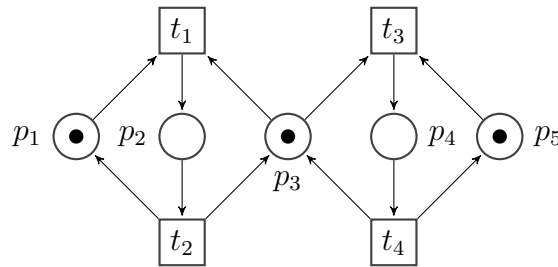


## Exercise Sheet 5

### Problem 1: Unfolding Prefix

Use the ERV algorithm given in class with McMillan's adequate order to compute the finite and complete prefix of the unfolding of the following Petri net.



Provide the set of possible extensions and cut-offs at each iteration. *Note: The initial configuration of an unfolding can be seen as the local configuration  $[e_{\perp}]$  caused by an event  $e_{\perp}$ .*

### Problem 2: Adequate Orderings

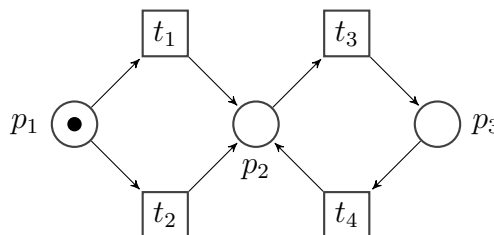
Establish adequacy of the following orderings:

- (a)  $\prec_m$ :  $[e] \prec [e']$  iff.  $||[e]|| < ||[e']||$ ;
- (b)  $\prec_{lex}$ :  $[e] \prec_{lex} [e']$  iff.  $||[e]|| <_1 ||[e']||$  or,  $||[e]|| = ||[e']||$  and  $\text{order}([e]) <_{lex} \text{order}([e'])$ .

Assume the set of transitions is totally ordered by the transitions' indices. Define  $\text{order}(C) := t_1^{\#t_1(C)} \dots t_n^{\#t_n(C)}$ , where  $\#_t(C)$  denotes the number of events  $e \in C$  labelled by  $t$ . For example, if  $C = \{e_2, e_4, e_3, e_1\}$  is labelled by  $h(e_2) = h(e_4) = h(e_3) = t_1, h(e_1) = t_3$  then  $\text{order}(E) := t_1^3 t_2^0 t_3^1 t_4^0 = t_1 t_1 t_1 t_3$ . Hence, for any two configurations  $C_1$  and  $C_2$ , the strings  $\text{order}(C_1)$  and  $\text{order}(C_2)$  can be ordered lexicographically by  $<_{lex}$  (e.g.  $t_1 t_1 t_1 t_3 <_{lex} t_1 t_2$ ).

### Problem 3: Yet Another Unfolding Prefix

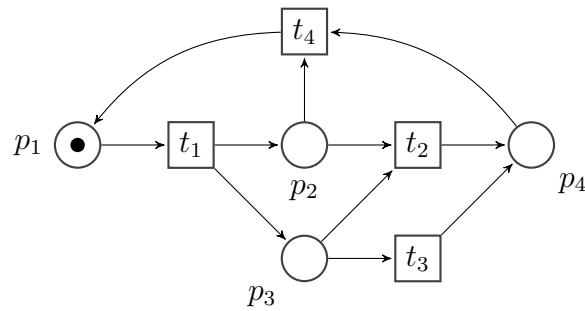
Consider the following Petri net:



Use  $\prec_m$  and  $\prec_{lex}$  from the previous problem to construct the net's unfolding. Provide the set of possible extensions and cut-offs at each iteration. What do you observe?

## Problem 4: SAT-Based Verification

Consider the Petri net depicted below.



(a) Compute the finite complete prefix of the Petri net's unfolding under McMillan's ( $\prec_m$ ) order and express the reachability of the marking  $(0\ 0\ 0\ 1)^T$  as violation constraint  $\mathcal{V}$ .

(b) Find a general formula  $\mathcal{M}$  characterizing reachability of a marking in a complete prefix starting from the  $\mathcal{C} \wedge \mathcal{V}$  formula given in class.

*Hint: Use extra Boolean variables  $x_b$  for every  $b \in B$  of the unfolding  $(\mathcal{O}, h) = (B, E, G)$ .*