Übungen zur Vorlesung
Concurrency Theory
Blatt 4

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Abgabe bis 18.06.2024 um 23:59 Uhr

Aufgabe 4.1 (Intersection non-emptiness of VASS reachability languages)

Let V_1, V_2 be two initalized VASS over Σ. The intersection non-emptiness problem asks whether $L(V_1) \cap L(V_2) \neq \emptyset$ holds. Show that intersection non-emptiness is decdiable for VASS reachability languages.
Note: This is the first step in the procedure that decides regular separability.

Aufgabe 4.2 (VASS languages are closed under rational transductions)

Let V be an initalized VASS over Σ and $T \subseteq \Sigma^* \times \Gamma^*$ a rational transduction between Σ and Γ. Show that the language $T(L(V)) = \{T(w) \mid w \in L(V)\}$ is a VASS reachability language.

Aufgabe 4.3 (Büchi boxes)

Let $A = (Q, \Sigma, q_0, \delta, Q_F)$ be an NFA over an alphabet Σ. The transition equivalence relation $\sim_A \subseteq \Sigma^* \times \Sigma^*$ is defined by $v \sim_A w$ if for all $q, q' \in Q$ we have $q \xrightarrow{v} q'$ iff $q \xrightarrow{w} q'$.

(a) Show that $\sim_A$ is an equivalence relation with finitely many equivalence classes.

(b) Show that for every word $w \in \Sigma^*$ the equivalence class $[w]_{\sim_A} := \{v \in \Sigma^* \mid v \sim_A w\}$ is a regular language.

Hint: First show that $L_{q,q'} := \left\{w \in \Sigma^* \mid q \xrightarrow{w} q'\right\}$ *is regular.*

Aufgabe 4.4 (The Ackermann function)

The three-argument Ackermann function φ is defined recursively as follows.

$$\begin{array}{lcll} \varphi\colon \mathbb{N}^3 \to \mathbb{N} & & & \\ \varphi(m,n,0) & = & m+n & \\ \varphi(m,0,1) & = & 0 & \\ \varphi(m,0,2) & = & 1 & \\ \varphi(m,0,x) & = & m & \text{for } x > 2 \\ \varphi(m,n,x) & = & \varphi(m,\varphi(m,n-1,x),x-1) & \text{for } n > 0 \text{ and } x > 0 \end{array}$$

(a) Formally prove the following equalities using induction:

$$\varphi(m,n,0) = m+n, \qquad \varphi(m,n,1) = m \cdot n, \qquad \varphi(m,n,2) = m^n.$$

(b) Nowadays, one usually considers the following two-parameter variant.

$$\begin{array}{lcll} A\colon \mathbb{N}^2 \to \mathbb{N} & & & \\ A(0,n) & = & n+1 & \\ A(m,0) & = & A(m-1,1) & \text{for } m>0 \\ A(m,n) & = & A(m-1,A(m,n-1)) & \text{for } m>0 \text{ and } n>0 \end{array}$$

For example, we have

$$A(1,2) = A(0,A(1,1)) = A(0,A(0,A(1,0))) = A(0,A(0,A(0,1))) = A(0,A(0,2)) = A(0,3) = 4\,.$$

Similar to this computation, write down a full evaluation of $A(2,3)$.

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