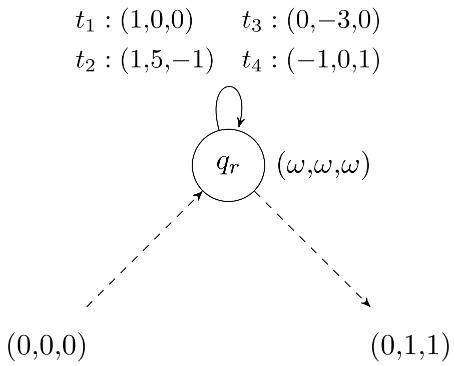
SS 2024 15.05.2024

Übungen zur Vorlesung
Concurrency Theory
Blatt 3

Prof. Dr. Roland Meyer
Jan Grünke Abgabe bis 28.05.2024 um 23:59 Uhr

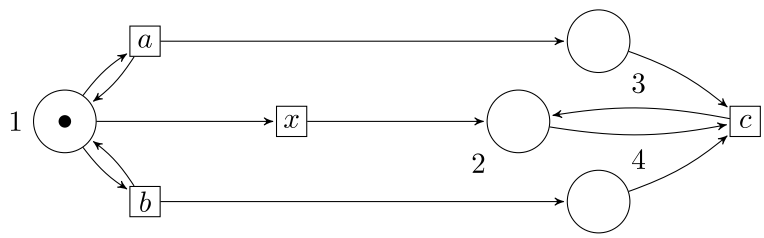
Aufgabe 3.1 (Decomposition)

Decompose the following MGTS into a set of perfect MGTS.



Aufgabe 3.2 (Rackoff's bound)

Consider the Petri net $N = (\{1,2,3,4\}, \{a,b,c,x\}, W)$ with multiplicities as depicted below. The initial marking of interest is $M_0 = (1,0,0,0)^T$ and the final marking is $M_f = (1,0,10,100)^T$.



Compute the values $m(3, M_0)$ and $f(3)$ and argue why they are correct.

Aufgabe 3.3 (Boundedness problem)

A Petri net is *bounded* if the set of reachable markings is finite, and *unbounded* otherwise. We consider the boundedness problem: Given a Petri net $N = (S, T, W, M_0)$ with initial marking of size n decide whether N is bounded. Similar to the coverability problem the boundedness problem is decidable using coverability graphs. In this excercise we want to show that it is decidable in EXPSPACE. Recall that a vector $M \in \mathbb{Z}^S$ is i-bounded if the first i counters are positive and i–r-bounded if the first i counters have values in $\{1, ..., r\}$. We call a sequence of vectors $M_1, ..., M_k$ in $\mathbb{Z}^S$ *self-covering* if there is some $j \in [1, k]$ such that $M_j < M_k$ (where we write $M_1 < M_2$ if $M_1 \leq M_2$ and $M_1 \neq M_2$). A run is self-covering if the marking sequence of it is self-covering.

(a) Show that N is unbounded iff there is a self-covering run. More precisely, show that N is unbounded iff there are transition sequences $\sigma_1, \sigma_2 \in T^*$ such that $M_0[\sigma_1\rangle M_1[\sigma_2\rangle M_2$ and $M_1 < M_2$.

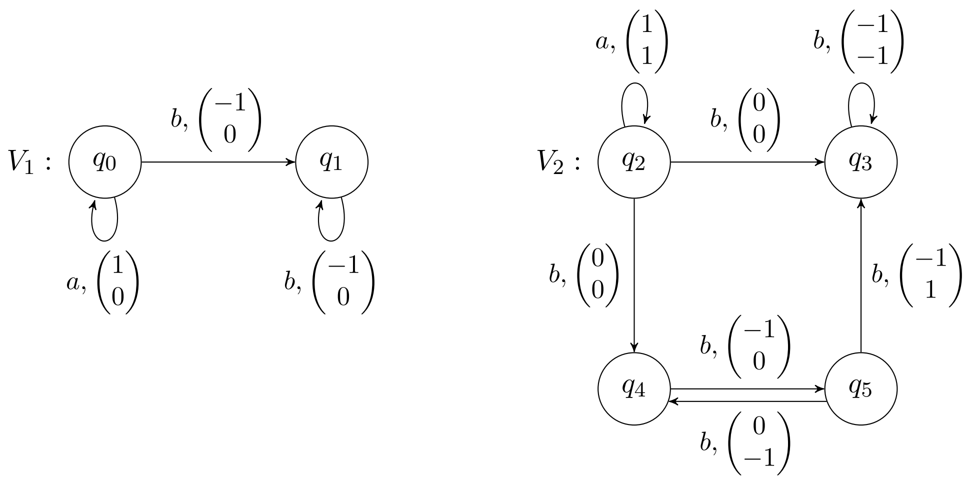
(b) For all $r > 1$ and $0 \leq i \leq |S|$ one can show the following: if there is a i–r-bounded self-covering run, then there is a short i–r-bounded self-covering run of length at most r^{n^c} for some constant c.

Use this to show $g(i+1) \leq (2^n g(i))^{n^c}$ with

$$m(i, M) := min\left\{|\sigma| \mid \sigma \in \mathbb{Z}^{S^*} \text{ is } i\text{-bounded self-covering run from } M\right\}$$
$$g(i) := max\left\{m(i, M) \mid M \in \mathbb{Z}^S\right\}.$$

Give a bound for $g(0)$.

Aufgabe 3.4 (Transducer Trick)
Let $(V_1, (q_0, (0,0)^T)), (q_1, (2,0)^T))$ and $(V_2, (q_2, (1,0)^T)), (q_3, (1,0)^T))$ be two initalized VASS over the alphabet $\Sigma = \{a, b\}$ where V_1 and V_2 are given by:



Give another VASS V such that $L(V)$ is regular separable from D_2 (Dyck language with two counters) if and only if $L(V_1)$ is regular separable from $L(V_2)$.

Abgabe bis 28.05.2024 um 23:59 Uhr per Mail an jan.gruenke1@tu-bs.de.