

## Exercise Sheet 12

### Problem 1: Greibach is hardest

Prove Proposition 3.5 from the lecture notes:

The Greibach language  $L_0$  is context-free.

### Problem 2: Flooding

Prove Proposition 4.1 from the lecture notes:

A language class is a full trio if and only if it is closed under

- homomorphisms
- intersection with regular sets, and
- flooding, which, for some  $a \in X$ , maps  $L \subseteq X^*$  to  $L \sqcup \{a\}^*$ .

### Problem 3: Grammars

Prove Proposition 4.2 from the lecture notes:

For each context-free grammar  $G = (N, T, P, S)$ , homomorphism  $\alpha : T^* \rightarrow U^*$ , and  $a \in T$ , one can construct context-free grammars  $G'$  and  $G''$  with  $L(G') = \alpha(L(G))$  and  $L(G'') = L(G) \sqcup \{a\}^*$ .

### Problem 4: Closure under Union

Prove Proposition 5.3 from the lecture notes:

Let  $\mathcal{C}$  be a finitely generated full trio, i.e. generated by a finite set of languages. Then,  $\mathcal{C}$  is a principal full trio if and only if  $\mathcal{C}$  is closed under union.