Exercises to the lecture
Concurrency Theory
Sheet 5

Roland Meyer, Viktor Vafeiadis Delivery until 27.05.2014 at 12h

Exercise 5.1 (Well-quasi orderings)

a) Prove or disprove that $(\mathbb{N}, |)$ is a well-quasi ordering where $a|b$ means "a divides b".

b) Let $(A, \leq)$ be a wqo. Prove that for $k \in \mathbb{N}$, $(A^k, \leq^k)$ is a wqo.

The ordering $\leq^k$ is obtained by component-wise application of $\leq$ on vectors in A^k. Thus, $(a_1, \ldots, a_k) \leq^k (a'_1, \ldots, a'_k)$ if and only if $a_i \leq a'_i$ for $i \in \{1, \ldots, k\}$.

c) Given a set A and a set $W \subseteq \mathbb{P}(A \times A)$, so that (A, w) is a wqo for all $w \in W$. Show that $(A, (\bigcup_{w \in W} w)^+)$ is a wqo.

Exercise 5.2 (Upward-closed sets)

a) For a finite alphabet Σ and $w_1, w_2 \in \Sigma^*$, let $w_1 \leq w_2$ if and only if w_1 is a subword of w_2 as defined in the lecture.

Show that for any language $\mathcal{L} \subseteq \Sigma^*$, the languages $\mathcal{L}\uparrow$ and $\mathcal{L}\downarrow$ are regular.

b) Let $(A, \leq)$ be a wqo and $M_1, M_2 \subseteq A$ finite. Show that it is decidable if $M_1\uparrow = M_2\uparrow$.

Exercise 5.3 (Well-Structured transition systems)

a) Consider a transition system $(\Gamma, \gamma_0, \rightarrow)$ and a relation $\leq \subseteq \Gamma \times \Gamma$. Prove that $\leq$ is a simulation if and only if $pre(I)$ is upward-closed for every upward-closed set $I \subseteq \Gamma$.

b) Let $TS = (\Gamma, \gamma_0, \rightarrow, \leq)$ be a well-structured transition system where

- $\gamma \leq \gamma'$ is decidable for all $\gamma, \gamma' \in \Gamma$ and
- for all $\gamma \in \Gamma$, the set $\text{post}(\gamma) := \{\gamma' \in \Gamma \mid \gamma \rightarrow \gamma'\}$ is finite and computable.

Prove that termination is decidable for TS.

Note: A transition system is *terminating*, if there are no infinite runs.

Exercise 5.4 (Petri nets)

Consider the following definition of *Petri nets* and their firing relation:

- A *Petri net* is a triple $N = (P, T, W)$ where P is a set of *places*, T is a set of *transitions*, and $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a *weight function.*
- A *marking* $M \in \mathbb{N}^{|P|}$ of N is a function that maps places to natural numbers.
- A transition $t \in T$ is *enabled* in M, if $M \geq W(-, t)$.

  $W(-, t)$ is the vector $(W(p_1, t), \ldots, W(p_{|P|}, t))$ and $W(t, -)$ is defined analogously.
- If t is enabled in M_1, it transforms the marking into a new marking M_2 by removing $W(-, t)$ from M and adding $W(t, -)$.

  Formally, the *firing relation* $[\rangle \subseteq \mathbb{N}^{|P|} \times T \times \mathbb{N}^{|P|}$ contains the triple (M_1, t, M_2) (or $M_1[t\rangle M_2$) if t is enabled in M_1 and $M_2 = (M_1 - W(-, t)) + W(t, -)$.

Given an *initial marking* M_0, prove that the transition system $(N^{|P|}, M_0, [\rangle, \leq^{|P|})$ is a well-structured transition system.

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