Exercises to the lecture
Complexity Theory
Sheet 2

Prof. Dr. Roland Meyer
Sören van der Wall Delivery until 16.11.2021 at 23:59

Exercise 2.1 (Non-Emptiness of Context-Free Languages)

We consider the following problem:

Non-Emptiness of Context-Free Langauges (CFL Non − Empty)	
Input:	A context-free grammar G in Chomsky normal form.
Question:	Is $L(G)$ non-empty?

Show that CFL Non − Empty is P-complete with respect to logspace-many-one reductions.

Hint: You may reduce from CVP for the hardness.

Exercise 2.2

In this exercise, we want to show the NP-completeness of the following problem:

Triple Path Cover (TPC)	
Input:	A directed graph G.
Question:	Can we cover G with three disjoint paths?

More precise, TPC asks whether there are three paths

$$v_1^{(1)} \to v_2^{(1)} \to \cdots \to v_{n_1}^{(1)}$$
$$v_1^{(2)} \to v_2^{(2)} \to \cdots \to v_{n_2}^{(2)}$$
$$v_1^{(3)} \to v_2^{(3)} \to \cdots \to v_{n_3}^{(3)}$$

without repeating vertices such that each vertex v of G appears as a $v_j^{(i)}$ in exactly one of the paths.

Show that TPC is NP-complete. The hardness should be established by a reduction from the well-known NP-complete problem of finding a *Hamiltonian Cycle:*

Hamiltonian Cycle (Hamil Cycle)	
Input:	A directed graph G.
Question:	Is there a cycle in G (without repetition) that visits all vertices?

Exercise 2.3 (Safe Petri Nets)
Consider the following definition:

- A **Petri Net** is a triple $N = (P, T, W)$, where $P = \{p_1, \ldots, p_{|P|}\}$ is a finite set of **places**, T is a finite set of **transitions** and $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a **weight function**.
- A **marking** of N is a map $M \in \mathbb{N}^{|P|}$ that maps places to natural numbers. Intuitively, a marking represents the number of *tokens* in all places.
- A transition t is **enabled** in a marking M if $M \geq W(-, t)$, where $W(-, t)$ denotes the vector $(W(p_1, t), \ldots, W(p_{|P|}, t))$. The vector $W(t, -)$ is defined similarly.
- If t is enabled in M, the transition can be **fired**: we obtain a new marking M' by subtracting $W(-, t)$ and adding $W(t, -)$. More formally, we write: $M \xrightarrow{t} M'$ if t is enabled in M and $M' = M - W(-, t) + W(t, -)$.
- If $\sigma = \sigma_1 \ldots \sigma_\ell$ is a sequence of transitions we also write $M \xrightarrow{\sigma} M'$ if there are markings $M_1, \ldots, M_{\ell+1}$ so that $M_1 = M$, $M_{\ell+1} = M'$ and $M_i \xrightarrow{\sigma_i} M_{i+1}$ for $i = 1, \ldots, \ell$.
- A marking M' is **reachable** from a marking M if there is a sequence of transitions σ so that $M \xrightarrow{\sigma} M'$.
- The Petri Net N is called **safe** from marking M if all markings reachable from M are in $\{0, 1\}^{|P|}$.

We are interested in the following problem.

Reachability for safe Petri Nets (Safe Reach)	
Input:	A Petri Net N, markings M, M' such that N is safe from M.
Question:	Is M' reachable from M?

Show that Safe Reach is PSPACE-complete.

Hint: Do not reduce QBF to Safe Reach. Pick an arbitrary problem in PSPACE, a problem decided by a polynomial-space-bounded TM and reduce it to Safe Reach. The cells of the TM's tape should then be simulated by places, the TM's transition relation gets simulated by the PN's transitions.

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