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Exercises to the lecture
Complexity Theory
Sheet 1

Prof. Dr. Roland Meyer
Sören van der Wall

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Exercise 1.1 (Turing Machine)

Let $\Sigma = \{a, b\}$ be the alphabet consisting of the letters a and b and w a word in Σ^*. We denote by $|w|_a$ the number of a's in w.
Consider the language $A = \{w \in \Sigma^* \mid |w|_a \geq \lfloor \frac{|w|}{2} \rfloor\}$. Construct a deterministic logspace-bounded Turing Machine M such that M accepts a word w if and only if $w \in A$. We also write $L(M) = A$. This shows that A is a member of the class Ł.

Hint: The machine M has one read-only input tape and several work tapes. The space consumption of M is the maximal space used on one of the work tapes.

Exercise 1.2 (Reductions and hardness)

Let A be a problem over Σ, formally a subset of Σ^* (a language). We define the *co-problem* of A to be $\bar{A} = \Sigma^* \setminus A$. Now let C be a complexity class. Then the *co-class* coC is the set of all co-problems of problems in C. Formally, $\mathsf{coC} = \{A \mid \bar{A} \in \mathsf{C}\}$.
Let R be any set of functions and assume that A is C-complete with respect to R-many-one reductions. Show that $\bar{A}$ is coC-complete with respect to R-many-one reductions.

Exercise 1.3 (Completeness in Ł)

Let Σ be a finite alphabet. Prove the following two statements:

a) A problem A over Σ is in Ł if and only if $A \leq_m^{log} \{0, 1\}$.

b) Any $A \in$ Ł with $A \neq \emptyset$ and $A \neq \Sigma^*$ is Ł-complete wrt. logspace-many-one reductions.

Exercise 1.4 (Acyclic reachability)

Consider the problem

Acyclic Path (ACPATH)	
Input:	A directed acyclic graph $G = (V, E)$ and $s, t \in V$.
Question:	Is there a path from s to t?

Show that we can reduce PATH to ACPATH with a logspace-many-one reduction and conclude that ACPATH is NL-complete.

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