Exercises to the lecture
Complexity Theory
Sheet 6

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M.Sc. Peter Chini Delivery until 09.12.2015 at 12h

Exercise 6.1 (Reducing $ACYCLICPATH$ to $2SAT$)

Let G be an acyclic graph and s and t vertices of G. We construct a formula F in CNF as follows: for any edge $x \to y$, we add a clause $(\neg x \vee y)$. Moreover, we add the clauses (s) and $(\neg t)$. Show the following:

$$F \text{ is satisfiable } \Leftrightarrow \text{ there is no path from } s \text{ to } t \text{ in } G.$$

Exercise 6.2 (Counter automata)

Let Σ be a finite alphabet and A an k-counter two-way automaton over Σ.

a) The counters of A may take values in $\mathbb{Z}$. Construct an k'-counter two-way automaton A' such that:
   - A' simulates A, and
   - the counters of A' only take values in $\mathbb{N}$.

b) Assume that A has linearly bounded semantics and that the counters can only take values in $\mathbb{N}$. Construct a k'-head two-way finite automaton B that simulates A.

   This is implication (2) $\Rightarrow$ (3) of the theorem from the lecture about the equal expressiveness of logspace-bounded Turing machines, k-counter two-way automaton with linearly bounded semantics and k-head two-way finite atuomata.

Exercise 6.3 (Circuit Value Problem)

We have seen in the lecture that CVP is P-complete with respect to logspace-many-one reductions. Evaluating a Boolean formula is in L. Intuitively, one would reduce CVP to the corresponding problem for Boolean formulas as follows:
Replace

- $P_k = P_i \vee P_j$ by $P_k \leftrightarrow P_i \vee P_j$,
- $P_k = P_i \wedge P_j$ by $P_k \leftrightarrow P_i \wedge P_j$.

Show that this is **not** a logspace-many-one reduction.

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