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Exercises to the lecture
Complexity Theory
Sheet 11

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Exercise 11.1 (Treewidth of Cliques)
Let G be a graph and $(T, \{X_t\}_{t \in V(T)})$ a tree decomposition of G. Show that each clique in G is contained in a single bag of $(T, \{X_t\}_{t \in V(T)})$.

Derive that $tw(G) \geq \omega(G) - 1$, where $\omega(G)$ is the maximal size of a clique in G.

Hint: Let C be the set of vertices of a clique and let st be an edge in the tree T. Use the separation lemma to show that either $C \subseteq V_s$ or $C \subseteq V_t$, where $V_s = \bigcup_{u \in T_s} X_u$ and $V_t = \bigcup_{u \in T_t} X_u$. Like in the separation lemma, the trees T_s and T_t are obtained from removing the edge st from T.

Exercise 11.2 (Parameterized Reductions preserve FPT)
Let A, B be two languages. Prove the following statement: If there is a parameterized reduction from A to B and B is FPT, then also A is FPT.

Exercise 11.3 (A Parameterized Reduction)
Let G be a graph. A set $D \subseteq V(G)$ is called a *dominating set* of G if each vertex $v \in V(G)$ has a neighbor in D. Consider the following problem:

Dominating Set	
Input:	A graph G and an integer k.
Parameter:	$k \in \mathbb{N}$.
Question:	Does there exist a dominating set D of G of size at most k?

We want to reduce Dominating Set to Set Cover:

Set Cover	
Input:	Sets $(S_i)_{i \in [1..m]}$ over a universe $U = \bigcup_{i \in [1..m]} S_i$, and an $\ell \in \mathbb{N}$.
Parameter:	$\ell \in \mathbb{N}$.
Question:	Are there ℓ sets $S_{i_1}, \ldots, S_{i_\ell}$ from the family such that $U = \bigcup_{j \in [1..\ell]} S_{i_j}$?

Construct a parameterized reduction from Dominating Set to Set Cover. Prove the correctness of your construction.

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