

Exercises to the lecture
Complexity Theory
Sheet 5

Prof. Dr. Roland Meyer
Dr. Prakash Saivasan

Delivery until 29.11.2016 at 10h

Exercise 5.1 (Completeness in L)

Let Σ be a finite alphabet. Prove the following two statements:

- A language A over Σ is in L if and only if $A \leq_m^{\log} \{0, 1\}$.
- Any language A in L that satisfies $A \neq \emptyset$ and $A \neq \Sigma^*$ is already L-complete with respect to logspace-many-one reductions.

Exercise 5.2 (Acyclic reachability)

Show that we can reduce *PATH* to *ACYCLICPATH* with respect to logspace-many-one reductions. Conclude that $\overline{ACYCLICPATH}$ is NL-complete.

Exercise 5.3 (Reducing *ACYCLICPATH* to *2SAT*)

Let G be an acyclic graph and s and t vertices of G . We construct a formula F in CNF as follows: for any edge $x \rightarrow y$, we add a clause $(\neg x \vee y)$. Moreover, we add the clauses (s) and $(\neg t)$. Show the following:

$$F \text{ is satisfiable} \Leftrightarrow \text{there is no path from } s \text{ to } t \text{ in } G.$$

Exercise 5.4 (Counter automata)

Let Σ be a finite alphabet and A an k -counter two-way automaton over Σ .

- The counters of A may take values in \mathbb{Z} . Construct an k' -counter two-way automaton A' such that:
 - A' simulates A , and
 - the counters of A' only take values in \mathbb{N} .
- Assume that A has linearly bounded semantics and that the counters can only take values in \mathbb{N} . Construct a k' -head two-way finite automaton B that simulates A .

Delivery until 29.11.2016 at 10h into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23