Exercises to the lecture
Complexity Theory
Sheet 3

Prof. Dr. Roland Meyer
Dr. Prakash Saivasan Delivery until 15.11.2016 at 10h

Exercise 3.1 (Time and space constructible functions)

Let $log_2(n)$ denote the logarithm to the base 2. Show the following:

a) $log_2(n)$ is space constructible.

b) $log_2(n)$ is not time constructible.

Note that the definition of time/space constructible functions requires a Turing Machine that starts with the unary encoding of n and displays the result on a designated output tape when it enters the accept state.

Exercise 3.2 (A universal Turing Machine)

Construct a deterministic Turing Machine U with one input tape (read only) and one work tape so that on input $e\#x$, U computes $M(x)$, where M is the deterministic *1-tape* Turing Machine encoded in e. Show that U uses $\mathcal{O}(|e| \cdot s(n))$ space if M uses $s(n)$ space.

Exercise 3.3 (A non-deterministic Turing Machine)

Consider the 3-tape Turing Machine:

$$M = (Q, \{a, b\}, \{a, b, \$, \textvisiblespace\}, \$, \textvisiblespace, \delta, q_{init}, q_{accept}, q_{reject}),$$

where $Q = \{q_{init}, q_{run}, q_{accept}, q_{reject}\}$ and δ is given below.

a) Determine the language of M.

b) Show that M is not a decider.

c) What is needed to turn M into a decider ?

$$\left(q_{init}, \begin{pmatrix} \$ \\ \sqcup \\ \sqcup \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{init}, \begin{pmatrix} \$ \\ a \\ b \end{pmatrix}, \begin{pmatrix} S \\ R \\ R \end{pmatrix}\right) \text{ or } \left(q_{run}, \begin{pmatrix} \$ \\ \sqcup \\ \sqcup \end{pmatrix}, \begin{pmatrix} R \\ L \\ L \end{pmatrix}\right)$$

$$\left(q_{init}, \begin{pmatrix} \$ \\ \$ \\ \$ \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{init}, \begin{pmatrix} \$ \\ \$ \\ \$ \end{pmatrix}, \begin{pmatrix} S \\ R \\ R \end{pmatrix}\right)$$

$$\left(q_{run}, \begin{pmatrix} a \\ a \\ \star \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{run}, \begin{pmatrix} a \\ a \\ \star \end{pmatrix}, \begin{pmatrix} R \\ L \\ S \end{pmatrix}\right)$$

$$\left(q_{run}, \begin{pmatrix} b \\ \star \\ b \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{run}, \begin{pmatrix} b \\ \star \\ b \end{pmatrix}, \begin{pmatrix} R \\ S \\ L \end{pmatrix}\right)$$

$$\left(q_{run}, \begin{pmatrix} \sqcup \\ \$ \\ \$ \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{accept}, \begin{pmatrix} \sqcup \\ \$ \\ \$ \end{pmatrix}, \begin{pmatrix} S \\ S \\ S \end{pmatrix}\right)$$

$$\left(q_{run}, \begin{pmatrix} \sqcup \\ \circ \\ \star \end{pmatrix}\right) \overset{\delta}{\mapsto} \left(q_{reject}, \begin{pmatrix} \sqcup \\ \circ \\ \star \end{pmatrix}, \begin{pmatrix} S \\ S \\ S \end{pmatrix}\right),$$

where $\star$ and $\circ$ denote an arbitrary symbol but not both are allowed to display \$ at the same time.

Exercise 3.4 (Savitch's Theorem)

Get familiar with the details of the proof of Savitch's Theorem. You can find it in the updated handwritten notes on the website.

Delivery until 15.11.2016 at 10h into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23