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Exercises to the lecture
Complexity Theory
Sheet 2

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Delivery until 08.11.2016 at 10h

Exercise 2.1 (The language $COPY^k$ in space and time)

Let $\Sigma = \{a, b, \#\}$ be an alphabet. We define the language $COPY^k$ as follows:

$$COPY^k = \{w.\#.w.\# \dots \#.w.\#.w \mid w \in \{a,b\}^*,\ \#\text{ occurs } k \text{ times}\}.$$

Note that the language $COPY$ from the last exercise sheet is just $COPY^1$.

Show the following:

a) $COPY^k \in \text{DTIMESPACE}(\mathcal{O}(n), \mathcal{O}(n))$.

b) $COPY \in \mathsf{DSPACE}(\mathcal{O}(log\, n))$.

Recall the definition of DTIMESPACE *and note that there is an additional input tape.*

Exercise 2.2 (Complement classes)

Let $C \subseteq \mathbb{P}(\{0,1\}^*)$ be a complexity class. The complement class of C is defined as:

$$\text{co-}C = \{L \subseteq \{0,1\}^* \mid \bar{L} \in C\}.$$

a) Prove that if C is deterministic, we have: $C = \text{co-}C$.

b) Let I be an index set and $C_i, i \in I$ complexity classes. Show that the following equality holds:
$$\text{co-}\bigcup_{i \in I} C_i = \bigcup_{i \in I} \text{co-}C_i.$$

c) Deduce from the previous results that $P = \text{co-}P$.

Exercise 2.3 (Tape compression)

Show that for all $0 < \varepsilon \leq 1$ and all $s : \mathbb{N} \to \mathbb{N}$, we have:

$$\mathsf{DSPACE}(s(n)) \subseteq \mathsf{DSPACE}(\lceil \varepsilon \cdot s(n) \rceil).$$

Hint: Choose c to be $\lceil \frac{1}{\varepsilon} \rceil$. Then simulate a 1-tape Turing Machine M by a 1-tape Turing Machine M' with tape alphabet Γ^c. Encode a block of c cells of M into one cell of M'. Note that you have to remember the position of M's head inside such a block. How much space does M' use ?

Delivery until 08.11.2016 at 10h into the box next to room 343 in the Institute for Theoretical Computer Science, Muehlenpfordstrasse 22-23