Applied Automata Theory (WS 2013/2014)	Technische Universität Kaiserslautern

Exercise Sheet 8

Jun.-Prof. Roland Meyer, Georgel Calin Due: Tue, Dec 17

Exercise 8.1 König's Theorem

Let T be a finite set of square tiles with coloured borders. A tile cannot be rotated and two tiles can be joined when the joining border has the same colour.

A tiling of the first quadrant is a function $\sigma\colon \mathbb{N}\times\mathbb{N} \to T$ such that for all $(i,j) \in \mathbb{N}\times\mathbb{N}$ the tiles $\sigma(i,j)$ and $\sigma(i+1,j)$ can be joined horizontally, and the tiles $\sigma(i,j)$ and $\sigma(i,j+1)$ can be joined vertically. In other words, infinitely many copies of the tiles in T can cover the first quadrant while being joined properly.

Let $D_n := \{0,\dots,n\} \times \{0,\dots,n\}$ for any $n \in \mathbb{N}$. A tiling of $D_n \subseteq \mathbb{N}\times\mathbb{N}$ is defined similarly: the tiles have to be joined properly within D_n.

Use König's theorem to prove that if there is a tiling of every square D_n then there is a tiling for the first quadrant $\mathbb{N}\times\mathbb{N}$.

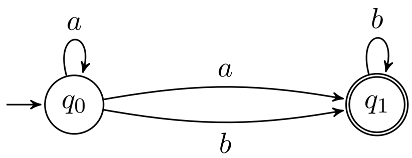
Hint: Order the tilings of the squares in a finitely-branching infinite tree.

Exercise 8.2 Variation of Ramsey's Theorem

Let (V,E) be an infinite graph such that for every infinite set of vertices $X \subseteq V$ there are $v, v' \in X$ with $(v,v') \in E$. Prove that (V,E) contains an infinite complete subgraph.

Exercise 8.3 NBA Complementation

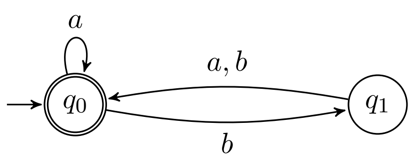
Consider the NBA A over $\Sigma = \{a,b\}$ below:



Use Büchi's complementation method discussed in class to compute $L(A)$ and $\overline{L(A)}$.

Exercise 8.4 NBA Universality

Consider the NBA A over $\Sigma = \{a,b\}$ below:



Use Fogarty & Vardi's method discussed in class to prove $L(A) = \Sigma^\omega$.