Applied Automata Theory (WS 2013/2014) Technische Universität Kaiserslautern

Exercise Sheet 4

Jun.-Prof. Roland Meyer, Georgel Calin Due: Tue, Nov 19

Exercise 4.1 "Presburger ⇒ NFA" Algorithm

(a) Prove the correctness of the construction given in class: *for every $q \in \mathbb{Z}$ and $w \in (\{0,1\}^n)^*$, the automaton accepts w starting from q iff. w encodes $\bar{c}$ with $\bar{a}\,\bar{c} \leq q$.*

(b) Construct a finite automaton for the atomic Presburger formula $x - 3y \leq 1$.

Exercise 4.2 "Presburger ⇒ NFA" for Atomic Formulas with Equality

One can modify the algorithm for $\bar{a}\,\bar{x} \leq b$ to produce an NFA for $\bar{a}\,\bar{x} = b$ by making the state $0 \in \mathbb{Z}$ the only accepting state and by updating the transition relation

$$\rightarrow := \rightarrow \cup \{q \xrightarrow{\overline{\text{bit}}} j\} \text{ only when } q - \bar{a}\,\overline{\text{bit}} \text{ is even.}$$

(a) Use the modified algorithm to construct a finite automaton for $x - 2y = 1$.

(b) Verify your result in (a) by checking that $L(A_{x-2y=1}) = L(A_{x-2y\leq 1}) \cap L(A_{-x+2y\leq -1})$.

Exercise 4.3 Quantifier Elimination

Eliminate the quantifiers of the following formula using the method described in class:

$$\neg\forall x.\, 3x < 2y \vee y < 2x.$$

Exercise 4.4 Semilinear Sets

(a) Prove that semi-linear sets are Presburger definable: for any semi-linear set $S \subseteq \mathbb{N}^n$ there exists a Presburger formula φ_S such that $S = Sol(\varphi_S)$.

(b) A function $f\colon \mathbb{N}^n \to \mathbb{N}^m$ is linear if $f(x+y) = f(x) + f(y)$ and $f(kx) = kf(x)$ for all $k \in \mathbb{N}$. Prove that semi-linear sets are closed under linear functions, i.e. if $S \subseteq \mathbb{N}^n$ is semi-linear and $f\colon \mathbb{N}^n \to \mathbb{N}^m$ is a linear function then $f(S) \subseteq \mathbb{N}^m$ is semi-linear.