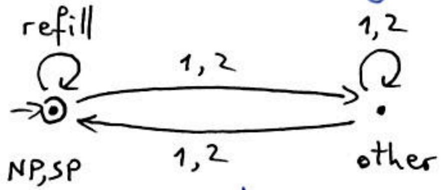
10.1 (a) In order not to run out of presents, the sleigh needs to be able to return to the poles from each continent at any time.

For this reason, since Asia is the continent which is „the farthest" from the poles, the sleigh is required to hold at least <u>4</u> presents when in Asia.

And since, in order to get to Asia, up to <u>4</u> presents are needed we conclude that the sleigh capacity needed equals <u>8</u> presents.

(b) A possible way to model A_{sleigh} is by



(c) An easy way to represent $\varphi_{Presents}$ is $\Box \Diamond$ refill

(d) The controller that guarantees Santa to never run out of presents will be the NBA that accepts

$$L(A_{Rudolf}) \cap L(A_{sleigh}) \cap L(\varphi_{Presents})$$

(e) By using predicates $p_{EU}, p_{NA}, \dots, p_{AS}$ to indicate when the sleigh enters a continent, we can intersect the language of the controller from (d) with

$$L(\Box\Diamond p_{EU} \wedge \Box\Diamond p_{NA} \wedge \dots \wedge \Box\Diamond p_{AS})$$

to obtain the language of a fair controller.

10.2.(a) $\varphi_1 = \Box(r \to \bigcirc s) \equiv \text{false}\ \mathcal{R}\ (\neg r \vee \bigcirc s)$

$\varphi_2 = \Box(c \to \Diamond r) \equiv \text{false}\ \mathcal{R}\ (\neg c \vee \text{true}\ \mathcal{U}\ r)$

$\varphi_3 = \Box(s \to (\neg r\ \mathcal{U}\ c)) \equiv \text{false}\ \mathcal{R}\ (\neg s \vee \neg r\ \mathcal{U}\ c)$

$\varphi_3' = \Box(s \to \bigcirc c) \equiv \text{false}\ \mathcal{R}\ (\neg s \vee \bigcirc c)$

(b) $FL(\varphi_1) = \{ \varphi_1, (\neg r \vee \bigcirc s) \wedge (\text{false} \vee \bigcirc \varphi_1), \neg r \vee \bigcirc s,$
$\text{false} \vee \bigcirc \varphi_1, \neg r, \bigcirc s, \text{false}, \bigcirc \varphi_1, s \}$

The initial states of the Vardi-Wolper constructed GNBA are the only reachable states since $\varphi_1 \in M$ implies that $\neg r \vee \bigcirc s \in M$ and $\text{false} \vee \bigcirc \varphi_1 \in M$, therefore $\bigcirc \varphi_1 \in M$.

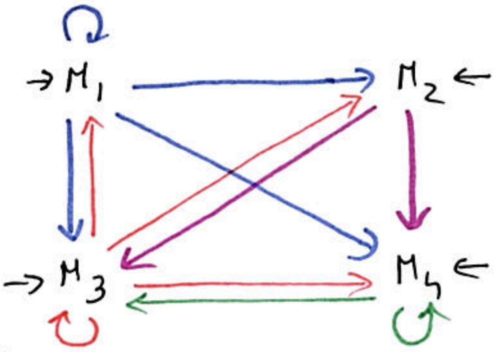
If $M_0 = \{ \varphi_1, (\neg r \vee \bigcirc s) \wedge (\text{false} \vee \bigcirc \varphi_1), \neg r \vee \bigcirc s,$ $\text{false} \vee \bigcirc \varphi_1, \bigcirc \varphi_1 \}$ then the minimal Hintikka states are

¬Hintikka

$M_1 = M_0 \cup \{\neg r\}$ $\qquad$ $M_3 = M_1 \cup \{s\}$
$M_2 = M_0 \cup \{\bigcirc s\}$ $\qquad$ $M_4 = M_2 \cup \{s\}$

The GNBA is depicted on the right with labels omitted:

$\xrightarrow{\{s\},\{c\}}$ $\quad$ $\xrightarrow{\{s\},\{r\},\{c\}}$ $\quad$ $\xrightarrow{\{s\}}$ $\quad$ $\xrightarrow{\{s\}}$



All states M_1, M_2, M_3, M_4 belong to the unique set of accepting states since there are no untils within φ_1.

The other formulas are similarly dealt with

(c) GNBAs $\qquad G_1 \qquad G_2 \qquad G_3 \qquad G_4$

$\qquad\qquad\quad \Updownarrow \qquad \Updownarrow \qquad \Updownarrow \qquad \Updownarrow$

NBAs $\qquad \underbrace{A_1 \cap A_2 \cap A_3 \cap A_4}$

this can be considered to be the overall automaton