8.1. Clearly $\mathbb{Z}^2$ is partially ordered by R where

$$sRs' \text{ iff. } s \xrightarrow{w} s' \text{ for some } w \in \{ \ell_a, \ell_b \}^+ .$$

Since each of T_1, T_2, T_3 is a well-founded partial order over $\mathbb{Z}^2$ and (by hypothesis) $R \subseteq T_1 \cup T_2 \cup T_3$, we can conclude from exercise 7.4 that R is well-founded.

This means that every infinite ordered sequence in R is stabilizing which proves the given program terminates.

8.2. (a) „⇒" $[u]_{\sim A} = [v]_{\sim A}$ means that for all $\varrho, \varrho' \in Q$

$\varrho \xrightarrow{u} \varrho'$ iff. $\varrho \xrightarrow{v} \varrho'$ and $\varrho \xrightarrow{u}_{fin} \varrho'$ iff. $\varrho \xrightarrow{v}_{fin} \varrho'$.

This implies that $R_{[u]_{\sim A}} = R_{[v]_{\sim A}}$ and $R^{fin}_{[u]_{\sim A}} = R^{fin}_{[v]_{\sim A}}$.

Therefore

$$\begin{aligned} Box(u) &= R_{[u]_{\sim A}} \underline{\cup} R^{fin}_{[u]_{\sim A}} \\ &= R_{[v]_{\sim A}} \underline{\cup} R^{fin}_{[v]_{\sim A}} = Box(v). \end{aligned}$$

„⇐" $Box(u) = Box(v)$ means $R_{[u]_{\sim A}} \underline{\cup} R^{fin}_{[u]_{\sim A}} = R_{[v]_{\sim A}} \underline{\cup} R^{fin}_{[v]_{\sim A}}$,

so $R_{[u]_{\sim A}} = R_{[v]_{\sim A}}$ and $R^{fin}_{[u]_{\sim A}} = R^{fin}_{[v]_{\sim A}}$.

This implies that $\varrho \xrightarrow{u} \varrho'$ iff. $\varrho \xrightarrow{v} \varrho'$ and $\varrho \xrightarrow{u}_{fin} \varrho'$ iff. $\varrho \xrightarrow{v}_{fin} \varrho'$, i.e. $[u]_{\sim A} = [v]_{\sim A}$.

(b) „⇒" If $(\varrho, \varrho') \in Box(uv)$ it means $\exists \varrho''$ s.t. $\varrho \xrightarrow{u} \varrho''$ and $\varrho'' \xrightarrow{v} \varrho'$ so $(\varrho, \varrho'') \in Box(u)$ and $(\varrho'', \varrho') \in Box(v)$.

Then, since $R;S = \{ (\varrho, \varrho') \in Q^2 \mid \exists \varrho''. (\varrho, \varrho'') \in R \text{ and } (\varrho'', \varrho') \in S \}$ it means that $Box(uv) \subseteq Box(u) ; Box(v)$.

„⇐" If $(\varrho, \varrho') \in Box(u); Box(v)$ it means $\exists \varrho''$ s.t. $(\varrho, \varrho'') \in Box(u)$ and $(\varrho'', \varrho') \in Box(v)$. This implies $\varrho \xrightarrow{u} \varrho''$ and $\varrho'' \xrightarrow{v} \varrho'$.

Therefore $\varrho \xrightarrow{uv} \varrho'$ and $(\varrho, \varrho') \in Box(uv)$ which means that $Box(u); Box(v) \subseteq Box(uv)$.

8.2. (c) The following algorithm computes all $\sim_A$ equivalence classes

```
resultSet = workSet = {Box(ε)} ∪ ⋃_{a∈Σ} {Box(a)}
while workSet ≠ ∅ do
    pop Box(v) from workSet
    for each Box(u) in resultSet do
        if Box(u) ; Box(v) ∉ resultSet then
            add Box(uv) to resultSet and workSet
        end if
    end for
end while
return resultSet
```

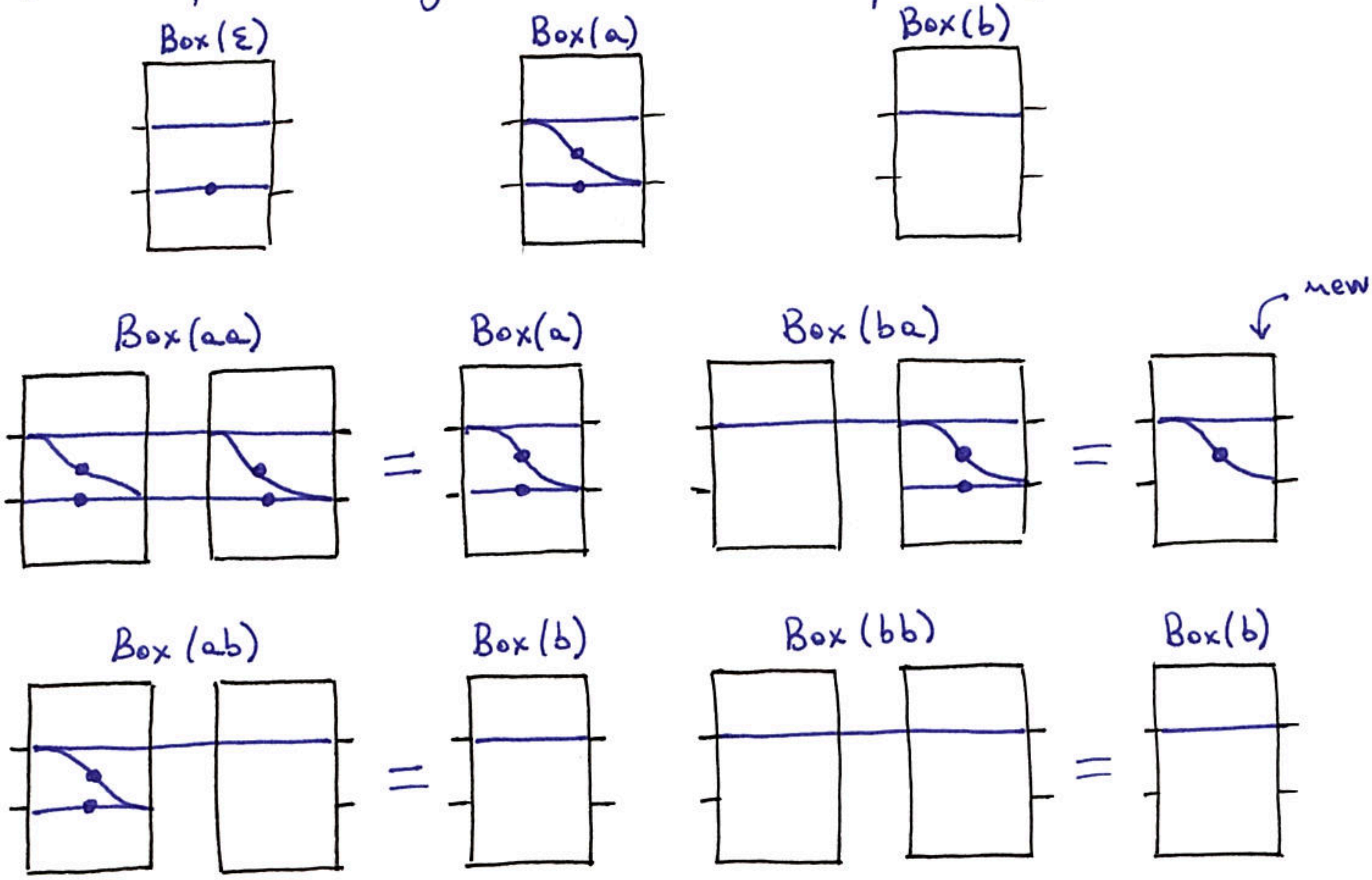
8.3. The following algorithm decides whether $uv^\omega \in L(A)$

```
for each q ∈ Q_A do
    if q_0 --uv--> q and ∃ n∈ℕ. q --v^n-->_fin q then
        return true
    end if
end for
return false
```

Modifying the 8.2.(c) algorithm by adding the following will decide whether $L(A) = \emptyset$.

```
for each Box(u) and Box(v) in resultSet do
    if uv^ω ∈ L(A) then
        return false
    end if
end for
return true
```

8.4. We first use algorithm 8.2.c to compute $\sim_A$



$\mathrm{Box}(ba\underline{b}) = \mathrm{Box}(bb)$
$= \mathrm{Box}(b)$

$\mathrm{Box}(ab\underline{a}) = \mathrm{Box}(ba)$

$\mathrm{Box}(ba\underline{ba}) = \mathrm{Box}(bba)$
$= \mathrm{Box}(ba)$

$\mathrm{Box}(b\underline{ba}) = \mathrm{Box}(ba)$

We then modify the algorithm for $L(A) = \emptyset$ in 8.3 as follows:

$\omega\text{-}\mathrm{Set}_{L(A)} = \omega\text{-}\mathrm{Set}_{\overline{L(A)}} = \emptyset$
for each $\mathrm{Box}(u)$ and $\mathrm{Box}(v)$ in resultSet do
  if $uv^\omega \in L(A)$ then add $([u]_{\sim A}, [v]_{\sim A})$ to $\omega\text{-}\mathrm{Set}_{L(A)}$
    else add $([u]_{\sim A}, [v]_{\sim A})$ to $\omega\text{-}\mathrm{Set}_{\overline{L(A)}}$
  end if
end for
return $\omega\text{-}\mathrm{Set}_{L(A)}$ and $\omega\text{-}\mathrm{Set}_{\overline{L(A)}}$

By applying the algorithm we get $L(A) = ([\varepsilon]_{\sim A} + [a]_{\sim A} + [b]_{\sim A} + [ba]_{\sim A}) [a]^\omega_{\sim A}$, respectively $\overline{L(A)} = ([\varepsilon]_{\sim A} + [a]_{\sim A} + [b]_{\sim A} + [ba]_{\sim A})([b]^\omega_{\sim A} + [ba]^\omega)$.