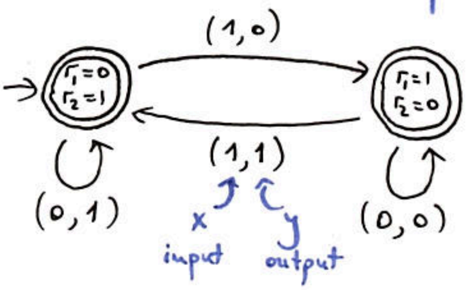
7.1. Notice the circuit computes $\quad r_1' = (x \oplus r_2) \vee (x \wedge r_1)$

$$y = r_2' = (x \oplus r_1) \vee (x \wedge r_2).$$

(a) The reachable part of the Büchi automaton accepting all infinite circuit runs then is $\underline{A}$:



(b) $A \models P_{fair}$ since $w \in L(A) \cap \{x \text{ infinitely often high}\}$ implies $w \in ((0,1)^*(1,0)(0,0)^*(1,1))^\omega$ so $w \in L(A) \cap \{y \text{ infinitely often high}\}$.

$A \not\models P_{safe}$ since e.g. $(0,1)^\omega \cancel{\notin} \in L(A) \setminus \{x=y=1 \text{ or } x=y=0\}$.

$A \not\models P_{persistent}$ since e.g. $(1,0)(0,0)^\omega \in L(A) \setminus \{\text{from some point, } y \text{ is high}\}$.

(c) E.g. $(1,1)^\omega \models P_{fair}$ $\quad$ and $\quad (1,0)^\omega \models \neg P_{fair}$

$(1,1)^\omega \models P_{safe} \qquad (1,0)^\omega \models \neg P_{persistent}$

$(1,1)^\omega \models P_{persistent} \qquad (1,0) \models \neg P_{safe}$.

7.2.(a) The idea could be to keep track of whether a process has been awaken by using binary „counters" and by accepting only when all counters have been „set".

If $\Sigma = \bigcup_{i=1}^{k} \{w(P_i), s(P_i)\}$, $Q = \{q_0\} \cup \bigcup_{i=1}^{k} \{P_i\}$, and $Q_F = Q$, then let $A_{MGF} = (\Sigma, Q \times \{0,1\}^k, (q_0, \underbrace{0,\dots,0}_{k\text{ times}}), \longrightarrow, \{(q_0, \underbrace{1,\dots,1}_{k\text{ times}})\})$ where „$\rightarrow$" is

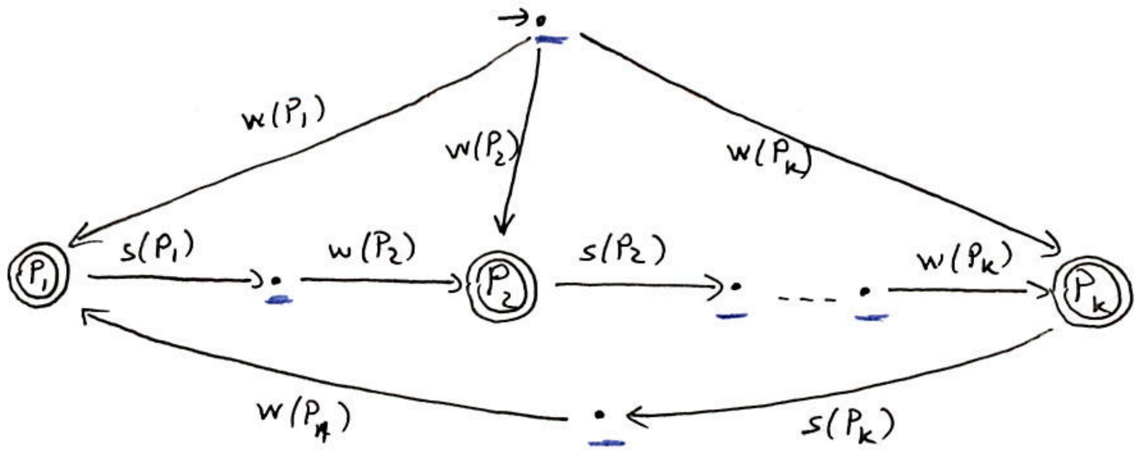
- $(q, \bar{c}) \xrightarrow{w(P_i)} (q', \bar{c})$ iff. $q \xrightarrow{w(P_i)} q'$ in A_{MG}
- $(q, \bar{c}) \xrightarrow{s(P_i)} (q', \bar{d})$ iff. $q \xrightarrow{s(P_i)} q'$ in A_{MG} and

either $\bar{c} = \bar{1}$ when $\bar{d} = e$;

or $\bar{c} \neq \bar{1}$ when $\bar{d} = \bar{c}[i \leftarrow 1]$

notation for substitution at position „i"

(b) The Büchi automaton for the Round Robin scheduling policy is:



We infeer that $L(A_{RR}) \subsetneq L(A_{MGF}) \subsetneq L(A_{MG})$.

(c) Since $L(A_{RR}) \subseteq L(A_{MGF})$ one can conclude

$$L(A_{OS}) \cap L(A_{RR}) \subseteq L(A_{OS}) \cap L(A_{MGF}),$$

hence if $L(A_{OS}) \cap L(A_{MGF}) \subseteq L(A_{Prop})$, then also

$$L(A_{OS}) \cap L(A_{RR}) \subseteq L(A_{Prop}).$$

7.3. (a) Since $w \in UV^{\omega}$, $w = u \cdot v_1 \cdot v_2 \cdot \ldots$ with $u \in U$ and $v_1, v_2, \ldots \in V$ and since $w \in L(A)$ there exists a run of A

$$s_0 \xrightarrow{u} s_1 \xrightarrow{v_1} s_2 \xrightarrow{v_2} s_3 \xrightarrow{v_3} \ldots \qquad (*)$$

which passes through accepting states infinitely often.

Let $w' = u' \cdot v_1' \cdot v_2' \cdot \ldots \in UV^{\omega}$ arbitrarily chosen. Since $u' \in U$, $v_i' \in V$ it means that also

$$s_0 \xrightarrow{u'} s_1 \xrightarrow{v_1'} s_2 \xrightarrow{v_2'} s_3 \xrightarrow{v_3'} \ldots \qquad (**)$$

is a run of A. Moreover, a final state is visited in (**) iff. ~~That~~ final state is visited in (*) by definition of equivalence classes wrt. $\sim_A$.

This implies that $w' \in L(A)$, therefore $UV^{\omega} \subseteq L(A)$.

7.3. (b) Assume $\exists w' \in UV^{\omega} \setminus \overline{L(A)}$ i.e. $w' \in UV^{\omega}$ and $w' \notin \overline{L(A)}$. Since $w' \notin \overline{L(A)} \iff w' \in L(A)$, by (a) we have $UV^{\omega} \subseteq L(A)$. Therefore, also $w \in L(A)$ since $w \in UV^{\omega}$, which is a contradiction to $w \in \overline{L(A)}$.

7.4. Let $f : A \times A \to \{1, .., n\}$ such that $f(a,b) = \min_{1 \le i \le n} \{ i \mid (a,b) \in T_i \}$.

Assume there is an infinite non-stabilizing sequence $(a_i)_{i \in \mathbb{N}}$ of R, i.e. $a_1 \gneq a_2 \gneq a_3 \gneq \dots$. Then, since in particular $(a_i)_{i \in \mathbb{N}} \subseteq A$, by Ramsey's theorem, there exists a monocromatic subsequence $(b_i)_{i \in \mathbb{N}}$ of $(a_i)_{i \in \mathbb{N}}$ which satisfies $b_1 \gneq b_2 \gneq b_3 \gneq \dots$ and $(b_i, b_j) \in T_k$ for a certain k.

However, the previous result contradicts well-foundedness of T_k, so the assumption was incorrect and R is well-founded.