# Advanced Automata Theory
# Exercise Sheet 11

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**Exercise 1: Yield of unranked trees**

Let $\Sigma$ be an unranked alphabet. We define the **yield** of a $\Sigma$-labeled tree inductively as

$$\text{yield}(a) = a \ ,$$
$$\text{yield}(b(t_1, \dots, t_n)) = \text{yield}(t_1) \dots \text{yield}(t_n) \ .$$

Intuitively, the yield of a tree $t$ is the finite word over $\Sigma$ that we get be reading the leaves of $t$ in-order (from left to right).

Let $\mathcal{L} \subseteq \Sigma^*$ be a regular language of finite words. Construct an NHA $A$ that accepts all trees whose yield is in $\mathcal{L}$,

$$\mathcal{L}(A) = \{t \colon \mathcal{T} \to \Sigma \mid \text{yield}(t) \in \mathcal{L}\} \ .$$

**Exercise 2: Examples of VPLs**

a) Construct a VPA for the so-called Dyck language

$$\mathcal{L}_a = \{w \in \{[, ]\}^* \mid w \text{ is well-bracketed}\} \ .$$

Argue that your VPA indeed accepts exactly this language as required.

*Example:* $[[][]] \in \mathcal{L}_a$ , $][ \notin \mathcal{L}_a$ .

b) Formally prove that the language of even-length palindromes

$$\mathcal{L}_b = \{ww^{\text{reverse}} \mid w \in \{a, b\}^*\}$$

is not a VPL language.

*Hint:* Consider the possible partitions of the alphabet. You may use that $\mathcal{L}_b$ is not regular.

**Exercise 3: Peeking at the top-of-stack**

According to our definition of VPAs, they are not allowed to make internal transitions dependent on the top-of-stack. We introduce a new automaton model VPA' that is allowed to peek at the top-of-stack at internal transitions

A VPA' $A$ is a tuple $A = (\Sigma^v, Q, Q_0, Q_F, \Gamma, (\delta_c, \delta_r, \delta_i))$, where all components are as in the definition of VPAs, but

$$\delta_i \subseteq Q \times \Sigma_i \times \Gamma \times Q \ .$$

A transition $q \xrightarrow{\text{check}\gamma}_a q' \in \delta_i$ can only be used if $\gamma$ is the topmost symbol on the stack, i.e. it induces the transition $(q, \sigma.\gamma) \to_a (q', \sigma.\gamma)$ on configurations.

Prove that for every VPA' $A'$, there is a language-equivalent VPA $A$.

*Hint:* Encode the top-of-stack into the control state, and use $\Gamma^2 \dot\cup \{\bot\}$ as stack alphabet.

**Exercise 4: Closure properties**

Let $\Sigma^v$ be a visibly alphabet. Prove that VPA-languages over $\Sigma^v$ are effectively closed under

a) union, and

b) intersection.

Given two VPAs $A_1, A_2$ over $\Sigma^v$, show how to construct VPAs for $\mathcal{L}(A_1) \cup \mathcal{L}(A_2)$ resp. $\mathcal{L}(A_1) \cap \mathcal{L}(A_2)$.