Advanced Automata Theory
Exercise Sheet 9

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Exercise 1: LTL to GNBA

Consider the following specification of your life: $\varphi := \Box(study\,\mathcal{U}\,exam)$

a) Translate φ into an equivalent formula φ' that uses only the basic definition of LTL.

b) Translate φ' into an equivalent formula φ'' in PNF.

c) Compute the Fisher-Ladner closure $FL(\varphi'')$.

d) Using the method from class, compute a GNBA A_φ with $L(A_\varphi) = L(\varphi)$.

Exercise 2: Past Time LTL I

LTL as presented in the lecture reasons about the future, i.e. positions right of the current one. We extend LTL to also reason about the past, i.e. positions left of the current one. The full syntax of Past Time LTL is as follows:

$$\varphi \quad ::= \quad p \mid \varphi \vee \psi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\psi \mid \underbrace{\circleddash\varphi}_{\text{"previous"}} \mid \underbrace{\varphi\,\mathcal{S}\,\psi}_{\text{"since"}}$$

The two new operators are defined as follows:

- $w, i \models \circleddash\varphi$ iff $i > 0$ and $w, i-1 \models \varphi$ *(i.e. the previous position satisfies φ)*
- $w, i \models \varphi\,\mathcal{S}\,\psi$ iff there is $k \leq i$ so that for all $k \leq j < i$ we have $w, j \models \varphi$, and $w, k \models \psi$ *(i.e. there is a position where ψ holds and since then φ was satisfied until now)*

We define LTL[...] as the LTL syntax restricted to $p, \vee, \neg$ and the operators given in brackets. Note that we can always use $\wedge$ and $\rightarrow$ because they can be expressed using the included operators.

a) Find LTL[$\circleddash, \mathcal{S}$] definitions for $\diamondminus$ and $\boxminus$ such that

1. $w, i \models \diamondminus\varphi$ iff there is $k \leq i$ with $w, k \models \varphi$
2. $w, i \models \boxminus\varphi$ iff $w, k \models \varphi$ holds for all $k \leq i$

b) The formula $\Box(ack \rightarrow \diamondminus req)$ specifies that every acknowledgment is preceded by an earlier request. Give an equivalent LTL[$\bigcirc, \mathcal{U}$] formula.

c) Kamp's theorem states the following:

> A language is FO[<] definable if and only if it is LTL[$\bigcirc, \mathcal{U}$] definable.

Using Kamp's theorem, show that a language is LTL[$\bigcirc, \mathcal{U}$] definable iff it is LTL[$\bigcirc, \mathcal{U}, \circleddash, \mathcal{S}$] definable.

Exercise 3: Past Time LTL II

Let $\models_{\text{fin}}$ be the satisfaction relation between **finite** words and LTL[$\bigcirc, \mathcal{U}, \ominus, \mathcal{S}$] formulas, defined exactly as the $\models$ relation but only considering positions of the word at hand. We further define $\mathcal{L}_{\text{fin}}(\varphi) = \{u \in \Sigma^* \mid u, 0 \models_{\text{fin}} \varphi\}$.

We want to show that properties of the shape $\Diamond\varphi$ where φ only talks about the past can only speak about finite prefixes (i.e. they are safety properties). Consider $\varphi \in$ LTL[$\ominus, \mathcal{S}$]:

a) Prove that, for every $w \in \Sigma^\omega$, every position $i \in \mathbb{N}$ and every $j \geq i$,

$$w, i \models \varphi \qquad \text{iff} \qquad w_0 \dots w_j, i \models_{\text{fin}} \varphi$$

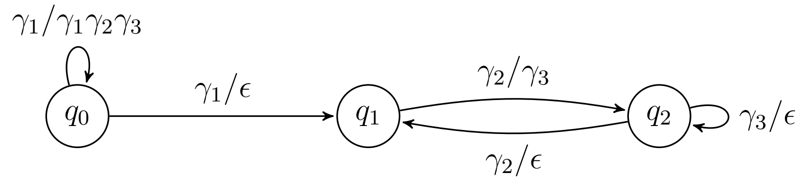
b) Prove $\mathcal{L}_{\text{fin}}(\Diamond\varphi)$ is a star-free language.

c) Show that $\mathcal{L}(\Diamond\varphi) = R \cdot \Sigma^\omega$ for some star-free language $R \subseteq \Sigma^*$.

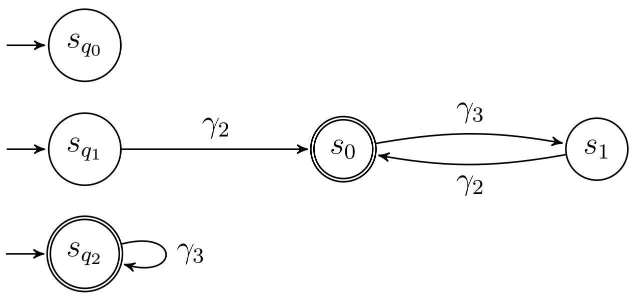
[*Hint: try with* $R = \mathcal{L}_{\mathit{fin}}(\Diamond\varphi)$]

Exercise 4: Pushdown Systems

Consider the following pushdown system P



And the following P-NFA A



Recall that CF(A) is the set of configurations (q, w) such that s_q accepts w in A:

a) Can P reach a configuration in CF(A) from $(q_1, \gamma_2\gamma_3\gamma_2\gamma_2\gamma_3\gamma_3)$?

b) Give a P-NFA A' with CF(A) $\cup$ pre(CF(A)) = CF(A').

c) Give a P-NFA B with CF(A) $\cup$ pre(CF(A)) $\subseteq$ CF(B) $\subseteq$ pre*(CF(A)) that has at most 5 states.