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Out: 27 April — Due: 2 May, 12:00

Exercise 1: Extensions of WMSO

a) Let us consider WMSO[<, suc, .], the set of WMSO-formulas extended by concatenation. This means if φ, ψ are WMSO[<, suc, .]-formulas, then $\varphi.\psi$ is also a WMSO[<, suc, .]-formulas. Give semantics to concatenation (i.e. define when $\mathcal{S}(w), \mathcal{I} \models \varphi.\psi$ should be satisfied) so that

$$\mathcal{L}(\varphi.\psi) = \mathcal{L}(\varphi).\mathcal{L}(\psi) .$$

b) Present a WMSO[<, suc] formula that is equivalent to $\varphi.\psi$.

c) For some fixed alphabet Σ, let us consider WMSO[<, suc, $[a]_{a\in\Sigma}$], the set of WMSO-formulas extended by an operator [a] for each symbol of the alphabet, i.e. if φ is a WMSO[<, suc, $[a]_{a\in\Sigma}$]-formula, then $[a]\varphi$ for any $a \in \Sigma$ is also a WMSO[<, suc, $[a]_{a\in\Sigma}$]-formula. Give semantics to $[a]\varphi$ so that

$$\mathcal{L}([a]\varphi) = \{w \in \Sigma^* \mid aw \in \mathcal{L}(\varphi)\} .$$

Furthermore, prove that WMSO[<, suc, $[a]_{a\in\Sigma}$] is equally expressive as WMSO[<, suc] by extending the translation form formulas to automata. This means that you should show how given an automaton for $\mathcal{L}(\varphi)$, one can construct an automaton for $\mathcal{L}([a]\varphi)$.

Exercise 2: Weak Dyadic Second Order Logic

Let WDSO be like WMSO with the modification that all second order variables X are dyadic instead of being monadic, i.e. they represent sets of pairs. Instead of having a predicate X(x) ("x is in X"), we have predicate X(x, y) ("(x, y) is in X"). The syntax and semantics of WDSO are the same with those of WMSO up to the predicate replacement:

$\mathcal{S}(w), \mathcal{I} \models X(x, y)$ iff. $(\mathcal{I}(x), \mathcal{I}(y)) \in \mathcal{I}(X)$
$\mathcal{S}(w), \mathcal{I} \models \exists X.\varphi$ iff. there is a finite set $M \subseteq D(w) \times D(w)$ such that $\mathcal{S}(w), \mathcal{I}[X \mapsto M] \models \varphi$.

We want to show that the class of languages representable by WDSO-formulas does strictly include than the set of regular languages.

a) Give (with arguments) a WDSO-formula that defines the language $\{a^n b^n \mid n \geq 0\}$.

b) Show how to translate a WMSO-formula ψ into a WDSO-formula that defines the same language.

Exercise 3: From WMSO **to Finite Automata**

a) Using the method presented in the lecture, construct a finite automaton that accepts the language defined by the formula

$$\varphi = \exists x \exists y \colon x < y \wedge P_a(x) \wedge P_a(y) \,.$$

b) Present a WMSO[<, suc]-formula that defines the language $((aa)^*b)^*$.

Exercise 4: WMSO **Expressiveness**

a) Show that WMSO[<, suc] and WMSO[suc] are equally expressive.

b) Show that WMSO[<, suc] and WMSO[<] are equally expressive.