Applied Automata Theory (SS 2011)	Out: Wed, June 8 Due: Wed, June 22

Exercise Sheet 8

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Exercise 8.1

Present LTL-formulae that express the following conditions:

(a) In the first point in time in which q holds, p holds as well.

(b) While q holds, p holds as well.

(c) Every time q holds, p has held at least once before.

(d) q holds only finitely often.

Exercise 8.2

Which of the following LTL-formulae are valid (i.e. are satisfied for every ω-word)? For those bi-implications that are not valid, which of the implications hold (if any)?

- $\Box\varphi \wedge \Box\psi \leftrightarrow \Box(\varphi \wedge \psi)$
- $\Diamond\varphi \vee \Diamond\psi \leftrightarrow \Diamond(\varphi \vee \psi)$
- $\Box\varphi \vee \Box\psi \leftrightarrow \Box(\varphi \vee \psi)$
- $\Diamond\varphi \wedge \Diamond\psi \leftrightarrow \Diamond(\varphi \wedge \psi)$
- $(\Box\varphi \rightarrow \Diamond\psi) \leftrightarrow \varphi\mathcal{U}(\psi \vee \neg\varphi)$
- $\Diamond\Box\varphi \leftrightarrow \Box\Diamond\varphi$
- $\Box\Diamond\Box\varphi \leftrightarrow \Diamond\Box\Diamond\varphi$
- $\Box\Diamond\Box\varphi \leftrightarrow \Diamond\Diamond\Box\varphi$

Exercise 8.3

Let $n \geq 1$, $\mathcal{P} = \{q_1, \ldots, q_n\}$ and $L_n = \{a_0 a_1 \cdots \mid \forall i = 1, \ldots, n : \exists j \in \mathbb{N} : q_i \in a_j\}$.

(a) Present LTL-formulae φ_n such that $L(\varphi_n) = L_n$ and $|\varphi|$ is linear in n.

(b) Describe NBAs A_n such that $L(A_n) = L_n$.

(c) Show that each NBA A with $L(A) = L_n$ has at least 2^n states. *Hint:* Construct 2^n words that lie in L_n such that the following holds: After a fixed number of steps in the accepting runs, A has to enter distinct states for these words, since otherwise, one can construct a word outside of L_n that is accepted by A.

Exercise 8.4

Consider the construction of NBAs for LTL-formulae due to Vari and Wolper. In the "$\supseteq$"-direction of the correctness proof, the case $\varphi\mathcal{R}\psi$ was described as "similar" in the lecture. Present this case in the same level of detail as the $\varphi\mathcal{U}\psi$ case.