Applied Automata Theory (SS 2011) Out: Wed, May 18 Due: Wed, May 25

Exercise Sheet 5

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Exercise 5.1

Show that every ω-regular language is accepted by an NBA. This amounts to solving the following tasks:

(a) For NBA A and B, describe how an NBA C can be obtained with $\mathsf{L}(C) = \mathsf{L}(A) \cup \mathsf{L}(B)$.

(b) Describe how, from an NFA A with $\mathsf{L}(A) \cap \Sigma^+ \neq \emptyset$, an NBA B can be constructed such that $\mathsf{L}(B) = \mathsf{L}(A)^\omega$.

(c) For each NFA A and NBA B, present a construction of an NBA C with $\mathsf{L}(C) = \mathsf{L}(A) \cdot \mathsf{L}(B)$.

Please note the slight change in the definition of ω-regularity in the lecture notes.

Exercise 5.2

Show that every language accepted by an NBA is ω-regular. *Hint:* The finite union will range over the set of final states.

Exercise 5.3

Show that, given an NBA A, it is decidable whether or not $\mathsf{L}(A) = \emptyset$.

Exercise 5.4

Let $A = (Q, q_0, \rightarrow, Q_F)$ be an NBA over Σ. A run

$$r \;=\; q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2 \xrightarrow{a_2} \cdots$$

is said to be *strongly fair* if for each state q that occurs infinitely often, each edge $q \xrightarrow{a} q'$, $q' \in Q$, $a \in \Sigma$, occurs infinitely often. The *strongly fair semantics for Büchi automata* assigns to A the language

$$\mathsf{SFL}(A) := \{w \in \Sigma^\omega \mid \text{there exists a strongly fair run in } A \text{ over } w\}.$$

Show that this semantics does not increase the expressive power of Büchi automata, i.e., show that for each NBA A, there is an NBA B such that $\mathsf{L}(B) = \mathsf{SFL}(A)$. Here, you may assume the fact that the class of languages accepted by NBA is closed under intersection. *Hint:* First, extend the alphabet so as to encode the edge in use for each step.