Applied Automata Theory (SS 2011) Out: Wed, May 11 Due: Wed, May 18

Exercise Sheet 4

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Exercise 4.1

In the lecture, a winning strategy for Duplicator was presented for $G_k(a^i, a^j)$, provided that $i \geq 2^k - 1$ and $j \geq 2^k - 1$. Show, again by induction on k, that for any $u, v, w \in \Sigma^*$, there is a winning strategy for Duplicator for $G_k(uv^i w, uv^j w)$, if $i \geq 2^k - 1$ and $j \geq 2^k - 1$. (Your proof need not be more detailed than the one in the lecture).

Exercise 4.2

We once again consider the winning strategy for Duplicator for $G_k(a^i, a^j)$ when $i \geq 2^k - 1$ and $j \geq 2^k - 1$. Explain why there is no winning strategy for Duplicator when $i < 2^k - 1$ and $j < 2^k - 1$. That is, *roughly* describe a winning strategy for Spoiler (neither a formal proof nor exact calculations are necessary here).

Exercise 4.3

Show that the following languages are star-free:

(a) $\{a\}^*\{b\}^*$,

(b) the set of words $w \in \{a, b\}^*$ that contain the subword ab as often as the subword ba.

Exercise 4.4

(a) Show that for every FO[<]-sentence φ and first-order variables x, y, there is a formula $\psi(x, y)$ such that

$$S_w, I \models \psi \quad \iff \quad S_{w_i \cdots w_j} \models \varphi,$$

for $1 \leq i \leq j \leq n$ and each word $w \in \Sigma^*$, $w = w_1 \cdots w_n$, $w_1, \ldots, w_n \in \Sigma$ and $I(x) = i$, $I(y) = j$. That is, ψ has two free variables x, y such that ψ is satisfied on a word w if and only if φ is satisfied on the subword between the positions x and y.

(b) Deduce from (a) that the class of FO[<]-definable languages is closed under concatenation. Hint: if $\varepsilon \in L_1$ and $\varepsilon \notin L_2$, then $w \in L_1 L_2$ iff $w \in L_2$ or w has a decomposition uv in proper factors with $u \in L_1$ and $v \in L_2$.

(c) Infer that every star-free language is FO[<]-definable.