Applied Automata Theory (SS 2011) Out: Thur, May 5 Due: Wed, May 11

Exercise Sheet 3

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Exercise 3.1

Using the method presented in the lecture, construct a finite automaton that accepts the language defined by the formula $\varphi = \forall x : (P_a(x) \rightarrow \forall y : (x < y \rightarrow P_b(y)))$.

Exercise 3.2

Suppose the logic WMSO$[<, \mathrm{suc}, \sqsubseteq]$ is obtained from WMSO$[<, \mathrm{suc}]$ by adding the predicate $\sqsubseteq$, which is interpreted as

$$S_w, I \models X \sqsubseteq Y \quad :\Longleftrightarrow \quad \text{for each } x \in I(X), \text{ there is a } y \in I(Y) \text{ such that } x \leq y.$$

Describe how the proof of Theorem Büchi II has to be changed to apply to the logic WMSO$[<, \mathrm{suc}, \sqsubseteq]$.

Exercise 3.3

Given an automaton A and a WMSO$[<, \mathrm{suc}]$-formula φ, the *model checking* problem asks whether every word accepted by A is satisfied by φ. If the latter condition holds, we write $A \models \varphi$. Show that the model checking problem can be reduced (in the sense of Turing reduction) to the problem of whether in a given finite automaton, one given state can be reached.

Exercise 3.4

Similar to $\exists$WMSO, we define *universal* WMSO, denoted by $\forall$WMSO, as the syntactic restriction of WMSO to formulas

$$\forall X_1 : \ldots \forall X_n : \varphi$$

where φ does not contain second-order quantification. Show that a language is regular iff it is $\forall$WMSO-definable.