Applied Automata Theory (SS 2011) Out: Wed, Apr 27 Due: Tue, May 3

Exercise Sheet 2

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Exercise 2.1

Since in the lecture, WMSO_0 has not been defined, we include a short explanation. In WMSO_0 for words, the signature contains the predicates $\subseteq$, Sing, Suc, and $\subseteq P_a$, which are interpreted such that

- $X \subseteq Y$ denotes the usual inclusion,
- $\text{Sing}(X)$ means $|X| = 1$, i.e., X is a singleton,
- $\text{Suc}(X, Y)$ means that X and Y are singletons and the element in Y is the successor of the element in Y,
- $X \subseteq P_a$ means that all positions in X contain an a.

Furthermore, a WMSO_0-formula contains no first order variables.

(a) Show that any $\text{WMSO}[<, \text{suc}]$-definable language is already WMSO[suc]-definable. (This amounts to expressin $<$ with the help of suc.)

(b) Show that any WMSO[suc]-definable language is already WMSO_0-definable.

Exercise 2.2

In the last exercise, we saw that, with respect to languages, WMSO_0 is equally expressive as $\text{WMSO}[<, \text{suc}]$. Since it is often desirable to have a logic with as few predicate symbols as possible, we would like to eliminate the Sing-predicate. Assume that $|\Sigma| \geq 2$ and that we only consider word structures.

1. Present a formula φ in WMSO_0 that does not utilize the Sing-predicate and that expresses emptiness of a set (i.e., φ has a a free second order variable X and is satisfied by an interpretation I if and only if I assigns X to the empty set).
2. Present a WMSO_0-formula φ that does not use the Sing-predicate and expresses the property of being a singleton. (Hint: Singleton sets have exactly two subsets.)

Exercise 2.3

(a) Present a $\text{WMSO}[<, \text{suc}]$-formula that defines the language

$$\{w \in \{a, b\}^* \mid |w| \text{ is divisible by } 3\}.$$

(b) Present a WMSO[<, suc]-formula that defines the language $\{aaa, bbb\}^*$.

(c) Show that, for each alphabet Σ, the language defined by the following formula is regular:

$$\begin{aligned}\exists X :&(\forall x : \forall y : \forall z : (X(x) \wedge X(y) \wedge x < z \wedge z < y) \rightarrow X(z))\\ &\wedge (\exists x : \exists y : (x < y \wedge X(x) \wedge X(y)))\\ &\wedge (\forall x : X(x) \rightarrow P_a(x)).\end{aligned}$$

Exercise 2.4

Use Büchi's construction from the lecture to determine a WMSO[<, suc]-formula that defines the language accepted by the following automaton: