Applied Automata Theory (SS 2011) Out: Wed, Apr 20 Due: Tue, Apr 26

Exercise Sheet 1

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Exercise 1.1

Show that for any given finite automaton A,

(a) the powerset construction yields a deterministic finite automaton that accepts $L(A)$.

(b) we have $L(A^*) = L(A)^*$.

Exercise 1.2

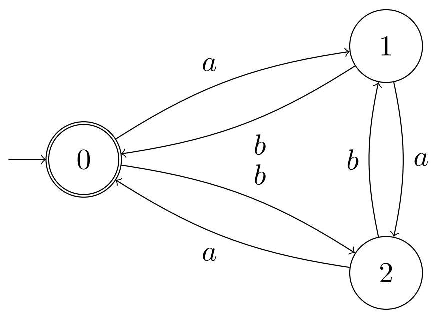
In the lecture, you learned about Arden's Lemma: *Let $U, V \subseteq \Sigma^*$ be languages such that $\varepsilon \notin U$. Then for any $L \subseteq \Sigma^*$, we have $L = UL \cup V$ if and only if $L = U^*V$.*

(a) Prove the "easy direction": Let $U, V, L \subseteq \Sigma^*$ be languages. Then $L = U^*V$ implies $L = UL \cup V$.

(b) In (a), we did not require $\varepsilon \notin U$. Show that $\varepsilon \notin U$ is necessary for the other direction to hold. Specifically, present sets U, V, and L such that $L = UL \cup V$, but $L \neq U^*V$. *Optional:* In the case $\varepsilon \in U$, can you describe the set of languages $L \subseteq \Sigma^*$ that satisfy $L = UL \cup V$?

Exercise 1.3

Use language equations and Arden's Lemma to determine a regular expression for the following finite automaton:



(The syntax used here defines initial states by an ingoing arrow (without a label), and final states have a double circle.)

Exercise 1.4

Show that, given finite automata A and B, it is decidable whether or not $L(A) = L(B)$.