Technische Universität Braunschweig

Institut für Theoretische Informatik

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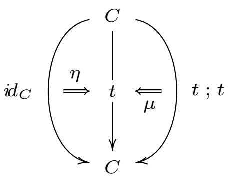
Algebraic Automata Theory

Sheet 1, 2017-10-27

Exercise 1 [10 Points]

The concept of a monad can be formulated in any 2-category $\mathscr{C}$, where the hom-functor lands in ***cat*** thather than ***set***, *i.e.*, $\mathscr{C}^{\mathrm{op}} \times \mathscr{C} \xrightarrow{\text{hom}}$ ***cat***: one considers a 1-cell $C \xrightarrow{t} C$ together with 2-cells



subject to the ususal associativity and unit conditions.

Identify the monads in the 2-category ***spn*** of spans:

- objects are sets;
- 1-cells $A \xrightarrow{\mathcal{S}} B$ are spans of functions $A \xleftarrow{s_0} S \xrightarrow{s_1} B$, generalizing directed graphs, which satisfy $A = B$ (alternatively, spans from A to B may be thought of as ***set*** valued $A \times B$ matrices);
- 2-cells from $A \xrightarrow{\mathcal{S}} B$ to $A \xrightarrow{\mathcal{T}} B$ are functions $S \xrightarrow{f} T$ making the two obvious triangles commute.

Exercise 2 [10 Points]

Identify the left adjoint 1-cells in the category ***spn*** (see above).

Exercise 3 [15 Points]

Describe a monad on the category ***set*** such that the EM-algebras are (abelian) groups.

due on Thursday, 2017-11-02, 13:15,