



## Algebraic Automata Theory

Sheet 0, 2017-10-22

### Exercise 1 [10 POINTS]

As we have seen, monoids precisely the are 1-object categories, while monoid-homomorphisms are precisely the functors between those. Characterize the natural transformations in this setting.

### Exercise 2 [10 POINTS]

Check that the family of functions

$$(\mathbb{P}(X))^* \xrightarrow{\delta_X} \mathbb{P}(X^*) \quad , \quad X \in \mathbf{set}$$

that map a string of  $n$  subsets  $A_i \subseteq X$  to their concatenation, *i.e.*, a subset of  $X^n \subseteq X^*$ , constitutes a distributive law between the free monoid monad and the power-set monad. What about a distributive law in the opposite direction?

### Exercise 3 [15 POINTS]

Consider  $\mathbb{RAT}$  as a functor on suitable category. Try to find a monad-structure on  $\mathbb{RAT}$ . How does this relate to other monads induced by  $\mathbb{F}$  and  $\mathbb{P}$ ?