Exercises to the lecture
Algorithmic Automata Theory
Sheet 4

Dr. Prakash Saivasan Delivery until 14.05.2019 at 15:00

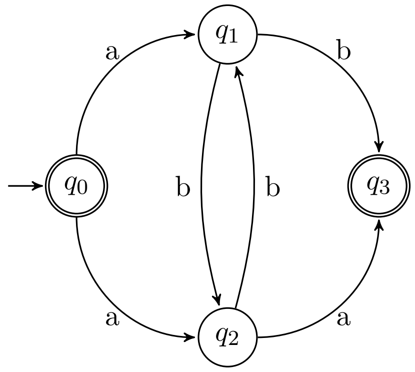
Exercise 4.1 (Finite Monoids)
Let M be a finite monoid. Prove the existence of an idempotent element in M: An element $t \in M$ such that $t \cdot t = t$.

Hint: Take an element $s \in M$ and consider the sequence $(s_i)_{i \in \mathbb{N}}$. Show that there are $i, p \in \mathbb{N}, p > 0$ such that $s^{m+p} = s^m$ for any $m \geq i$.

Exercise 4.2 (Transition Monoid)
Let $A = (\Sigma, Q, q_0, \delta, Q_F)$ be an NFA. Note that the transition relation can be seen as a function $\delta : Q \times \Sigma \to \mathcal{P}(Q)$. We define the *transition monoid* to be the set

$$M = \{\rho_x | x \in \Sigma^*, \rho_x(q) = \delta(q, x) \text{ for all } q \in Q\}.$$

Consider the automaton given below. Determine its transition monoid.



Exercise 4.3 (Monoid Operations)
Let L_1 and L_2 be regular languages.

- Show how to construct finite monoids recognising $L_1 \cup L_2$ from finite monoids accepting L_1 and L_2, what about $L_1 \cap L_2$?
- Show how to construct a finite monoid recognising $\bar{L}_1$ (compliment) from a finite monoid recognising L_1.

Delivery until 14.05.2019 at 15:00 into the box next to 343 or in the class